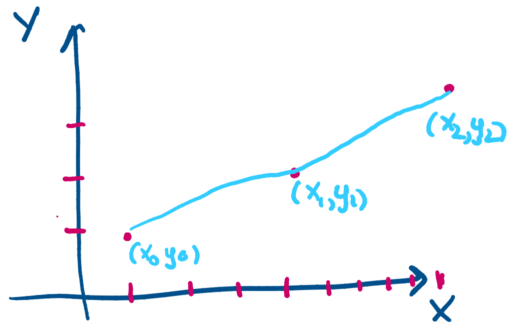
Online-Math4329

Tuesday, March 31, 2020 10:29 AM

$(1,1)$, $(4,2)$, $(9,3)$

$x_0\ y_0 \quad x_1\ y_1 \quad x_2\ y_2$



Newton's D.D formula

$$\boxed{\begin{aligned} P_2(x) &= f(x_0) + f[x_0,x_1](x-x_0) + \\ &\quad f[x_0,x_1,x_2](x-x_0)(x-x_1) \end{aligned}}$$

$f(x_0) = 1$ $\qquad f[x_0,x_1] = \dfrac{f(x_1) - f(x_0)}{x_1 - x_0}$

$$f[x_0,x_1,x_2] = \frac{f[x_1,x_2] - f[x_0,x_1]}{x_2 - x_0}$$

Table:

x_i	$f(x_i)$	$f[x_i, x_{i+1}]$	$f[x_0,x_1,x_2]$
$\boxed{1}$	1		
		$\frac{2-1}{4-1} = 1/3$	
4	2		$\frac{1/5 - 1/3}{9-1} = \frac{1}{8}\left(\frac{3-5}{15}\right) = \frac{-1}{60}$
		$\frac{3-2}{9-4} = 1/5$	
$\boxed{9}$	3		

$$P_2(x) = f(x_0) + f[x_0,x_1](x-x_0) + f[x_0,x_1,x_2](x-x_0)(x-x_1)$$

$$P_2(x) = 1 + 0.3333(x-1) + (-0.0167)(x-1)(x-4)$$

check: $\boxed{P_2(4) = 2}$

$$\begin{aligned} P_2(4) &= 1 + 0.3333(3) + (-0.0167)(4-1)(4-4) \\ &= 1 + 0.9999 - 0 \\ &\approx 1.9999 \approx 2 \end{aligned}$$

check: $\boxed{P_2(9) = 3}$

$$\begin{aligned} P_2(9) &= 1 + 0.3333(9-1) + (-0.0167)(9-1)(9-4) \\ &= 2.9984 \approx 3 \end{aligned}$$

$p_2(9) = 1 + 0.3333\,(9-1) + (-0.0167)($

$\quad = 2.9984 \approx 3$

Last video: $(1,1) \quad (4,2) \quad (9,3)$

$$p_2(x) = 1 + \underset{1/3}{0.3333}(x-1) - \underset{1/60}{0.0167}(x-1)(x-4)$$

Newton's formula:

$$p_2^N(x) = 1 + \frac{(x-1)}{3} - \frac{(x-1)(x-4)}{60}$$

easier to check with fractions!

$$p_2^N(1) = 1 + 0 = 1 \checkmark$$

$$p_2^N(4) = 1 + \frac{3}{3} - 0 = 2 \checkmark$$

$$p_2^N(9) = 1 + 8/3 - \frac{\overset{8}{(9-1)}\overset{5}{(9-4)}}{60} = 1 + 8/3 - 2/3 = 1 + 2 = 3 \checkmark$$

Towards Lagrange formula: $(1,1) \quad (4,2) \quad (9,3)$

$$p_2^L(x) = y_0 L_0(x) + y_1 L_1(x) + y_2 L_2(x)$$

$L_0, L_1, L_2 \rightarrow$ Lagrange Basis Functions

$y_0 = 1, \quad y_1 = 2, \quad y_2 = 3.$

$$p_2^L(x) = L_0(x) + 2L_1(x) + 3L_2(x)$$

How to construct $L_0(x)$, $L_1(x)$ & $L_2(x)$?

we want:

$L_0(x_0) = 1 \qquad L_0(x_1) = L_0(x_2) = 0$

$L_0(1) = 1 \qquad L_0(4) = L_0(9) = 0$

$L_1(x_0) = 0 \qquad L_1(x_1) = 1 \qquad L_1(x_2) = 0$

$L_1(1) = 0 \qquad L_1(4) = 1 \qquad L_1(9) = 0$

$L_2(x_0) = 0 \qquad L_2(x_1) = 0 \qquad L_2(x_2) = 1$

$L_2(1) = 0 \qquad L_2(4) = 0 \qquad L_2(9) = 1$

$$p_2^L(x) = L_0(x) + 2L_1(x) + 3L_2(x)$$

x_0

$L_0(1) + 2L_1(1) + 3L_2(1)$

$$P_2^L(x) = L_0(x) + 2L_1(x) + 3L_2(x)$$

$$\overset{x_0}{P_2^L(1)} = L_0(1) + 2L_1(1) + 3L_2(1)$$

$$= \underset{y_0}{1} + 2*0 + 3*0 = 1 \checkmark$$

$$\overset{P_2^L(x_1)}{P_2^L(4)} = L_0(4) + 2L_1(4) + 3L_2(4)$$

$$= 0 + 2*1 + 3*0 = 2 \checkmark$$

$P_2^L(9) = 3$ Do yourself!

$\hookrightarrow P_2^L(9) = L_0(\cancel{9})_0 + 2 * L_1(\cancel{9})_0 + 3 * L_2(\underline{9})^1 = 0 + 0 + 3*1 = 3$

$$P_2^L(x) = \overset{1}{y_0} L_0(x) + \overset{2}{y_1} L_1(x) + \overset{3}{y_2} L_2(x)$$

$$= L_0(x) + 2L_1(x) + 3L_2(x)$$

$L_0(x) \rightarrow L_0(x_0) = 1$ & $L_0(x_i) = 0$ $i = 1, 2.$

$$L_0(x) = (x - x_1)(x - x_2)$$

$$L_0(x_1) = (x_1 - x_1)(x_1 - x_2) = 0 = L_0(x_2)$$

$$= (x_2 - x_1)(x_2 - x_2)$$

$$L_0(x_0) = (x_0 - x_1)(x_0 - x_2)$$

but I want $L_0(x_0) = 1$ not $(x_0 - x_1)(x_0 - x_2)$.

scaling it!

$$L_0(x) = \frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)}$$

$$L_0(x_0) = \frac{(x_0 - x_1)(x_0 - x_2)}{(x_0 - x_1)(x_0 - x_2)} = 1$$

$$L_1(x) = (x - x_0)(x - x_2)$$

$L_1(x_0) = (x_0 - x_0)(x_0 - x_2) = 0$

$L_1(x_2) = (x_2 - x_0)(x_2 - x_2) = 0$

$L_1(x_1) = \underbrace{(x_1 - x_0)(x_1 - x_2)}_{\text{scaling factor}} \neq 1$

$$L_1(x_0) = L_1(x_2) = 0 \checkmark$$

but $L_1(x_1) \neq 1$ in general.

$$L_1(x_1) = (x_1 - x_0)(x_1 - x_2) \text{ not } 1!$$

$$L_1(x_1) = (x_1-x_0)(x_1-x_2) \text{ not } 1!$$

$$L_1(x) = \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}$$

$$L_1(x_1) = \frac{(x_1-x_0)(x_1-x_2)}{(x_1-x_0)(x_1-x_2)} = 1$$

$$L_2(x) = \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} \rightarrow \text{scaling factor.}$$

$L_2(x_0) = L_2(x_1) = 0$

$\underset{(1,1)}{x_0\,y_0}$ $\quad$ $\underset{(4,2)}{x_1\,y_1}$ $\quad$ $\underset{(9,3)}{x_2\,y_2}$

$$P_2^L(x) = y_0 L_0(x) + y_1 L_1(x) + y_2 L_2(x)$$

$$= y_0 \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} + y_1 \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} + y_2 \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}$$

$$P_2^L(x) = \frac{(x-\overset{A}{4})(x-\overset{B}{9})}{\underset{-3\ *\ -8}{(1-4)(1-9)}} + 2\frac{(x-1)(x-9)}{\underset{3\ *\ -5}{(4-1)(4-9)}} + 3\frac{(x-1)(x-4)}{\underset{8\ *\ 5}{(9-1)(9-4)}}$$

$$= \frac{x^2-13x+36}{24} + \frac{2(x^2-10x+9)}{-15} + \frac{3(x^2-5x+4)}{40}$$

used $(x-A)(x-B) = x^2-(A+B)x+AB$

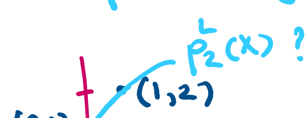
$$= \left(\frac{1}{24} - \frac{2}{15} + \frac{3}{40}\right)x^2 + \left(\frac{-13}{24} - \frac{20}{-15} - \frac{15}{40}\right)x +$$

$$\frac{36}{24} + \frac{18}{-15} + \frac{12}{40}$$ Simplify this!

$$P_2^L(x) = -\frac{1}{60}x^2 + \frac{5}{12}x + \frac{6}{10}$$

calculate the degree 2 poly $P_2^L(x)$ interpolating:

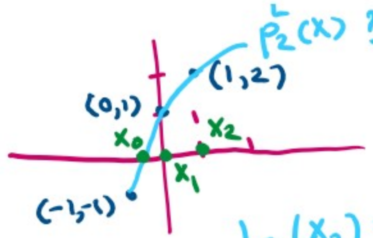
$(-1,-1)$ $\quad$ $\underset{x_1\ y_1}{(0,1)}$ $\quad$ $\underset{x_2\ y_2}{(1,2)}$



calculate the

$(-1,-1)$ $\quad (0,1)$ $\quad (1,2)$

$x_0\ y_0 \quad x_1\ y_1 \quad x_2\ y_2$



$$p_2^L(x) = \underset{-1}{y_0}L_0 + \underset{1}{y_1}L_1 + \underset{2}{y_2}L_2$$

$L_0(x_0)=1 \qquad L_1(x_1)=1 \qquad L_2(x_2)=1$

$$= -\frac{(x-x_1)(x-x_2)}{\underbrace{(x_0-x_1)(x_0-x_2)}} + \frac{(x-x_0)(x-x_2)}{\underbrace{(x_1-x_0)(x_1-x_2)}} + 2\frac{(x-x_0)(x-x_1)}{\underbrace{(x_2-x_0)(x_2-x_1)}}$$

scaling factors!

$$= -\frac{(x-0)(x-1)}{\underbrace{(\underset{-1}{-1-0})(\underset{-2}{-1-1})}_{2}} + \frac{(x-\overset{x_0}{(-1)})(x-1)}{\underbrace{(\underset{1}{0-(-1)})(\underset{-1}{0-1})}_{-1}} + 2\frac{(x-(-1))(x-0)}{\underbrace{(\underset{2}{1-(-1)})(1-0)}_{2}}$$

$$= \frac{-(x^2-x)}{2} + \frac{(x+1)(x-1)}{-1} + \frac{\cancel{2}(x+1)x}{\cancel{2}}$$

$$= \frac{-x^2+x}{2} + \frac{x^2-x+x-1}{-1} + x^2+x$$

$$= \frac{-x^2+x}{2} + (-1)(x^2-1) + x^2+x$$

$$= -\frac{x^2}{2} + \frac{x}{2} - x^2 + 1 + x^2 + x$$

$$= -\frac{x^2}{2} + \frac{x}{2} \cancel{-x^2}\cancel{+x^2} + 1 + x$$

$$p_2^L(x) = -\frac{x^2}{2} + \frac{x}{2} + x + 1 = \boxed{-0.5x^2 + 1.5x + 1}$$

check $p_2^L(-1)=-1$ $\quad p_2^L(0)=1$ and $p_2^L(1)=2$!

Show that $p_2^L(x)$ is the same as $p_2^N(x)$ obtained from using Newton's Divided difference formula.

x_i	$f(x_i)=y_i$	$f[x_i,x_{i+1}]$	$f[x_0,x_1,x_2]$

from using Newton's Divided difference formula.

x_i	$f(x_i)=y_i$	$f[x_i,x_{i+1}]$	$f[x_0,x_1,x_2]$
x_0 $\boxed{-1}$	-1 $f(x_0)$	$\frac{1-(-1)}{0-(-1)} = 2$	
0	1		$\frac{1-2}{1-(-1)} = \frac{-1}{2}$
x_2 $\boxed{1}$	2	$\frac{2-1}{1-0} = 1$	

$$p_2^N(x) = \overset{-1}{f(x_0)} + \overset{2}{f[x_0,x_1]}(x-x_0) + \overset{-0.5}{f[x_0,x_1,x_2]}(x-x_0)(x-x_1)$$

$$= -1 + 2(x-(-1)) + (-0.5)(x-(-1))(x-0)$$

$$= -1 + 2(x+1) + (-0.5)(x+1)x$$

$$= -1 + 2x+2 + -0.5x^2 - 0.5x$$

$$= -0.5x^2 + 2x - 0.5x + (-1) + 2 = \boxed{-0.5x^2 + 1.5x + 1}$$

Same!