Math 4329

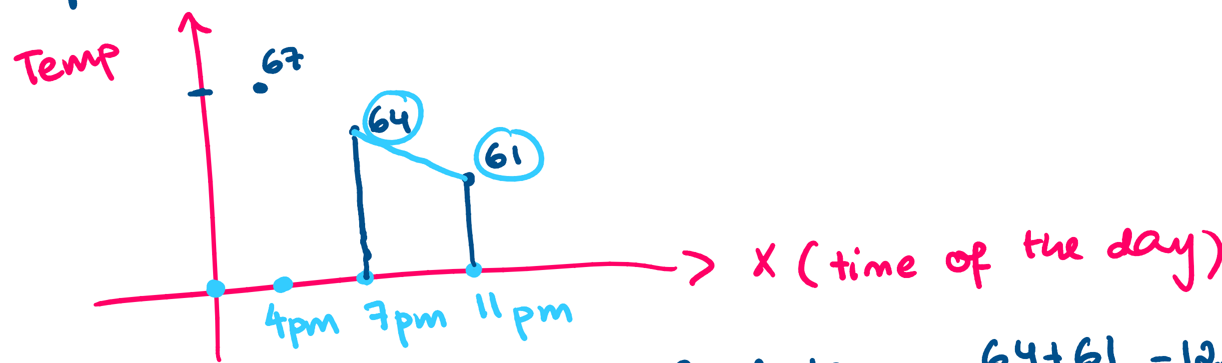
Thursday, March 12, 2020 4:41 PM

What will the temp. be at ~~8:30~~ 9:00 pm ?

Google: 67°F 64°F 61°F Temp

4pm 7pm 11pm Time

Drop the "units"



Temp at ~~8:30~~ 9:30 pm ? Average of temp $\frac{64+61}{2} = \frac{125}{2}$

$= 62.5$

Another way to fig. out temp at any time bet 7 & 11 pm is:

Straight line between $(7, 64)$ and $(11, 61)$

$(x_0\ \ y_0)$ $(x_1\ \ y_1)$

$y = mx + b$ $\quad f(x_0)$ $\quad f(x_1)$

$$p_1(x) = y_0 + \underbrace{f[x_0, x_1]}_{\text{slope of line } (x_0, y_0)\ \&\ (x_1, y_1)} (x - x_0)$$

$\dfrac{f(x_1) - f(x_0)}{x_1 - x_0}$ $\quad \Delta y / \Delta x$

$$p_1(x) = 64 + \underbrace{\frac{(61 - \overset{-3}{64})}{11 - 7_{\ 4}}}_{f[x_0, x_1]} (x - 7)$$

•(7,64)

•(11,61)

$\cdot(11, 61)$

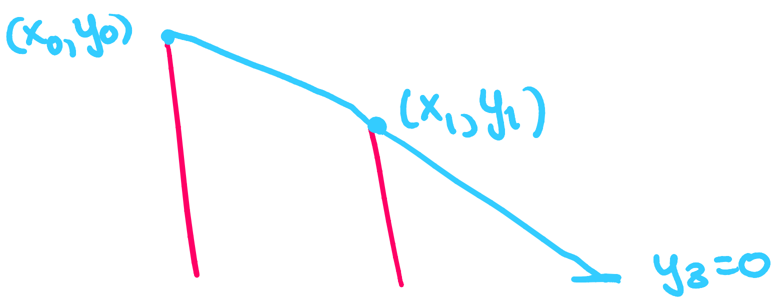
$$\boxed{p_1(x) = 64 + \underbrace{(-3/4)}_{f[x_0,x_1]}(x-7)}$$

formula represents temp. bet 7pm & 11 pm today.

$$p_1(9) = 64 + (-3/4)(9-7)$$
$$= 64 - 3/4(2)$$
$$= 64 - 1.5 = 62.5$$

$$\frac{7+11}{2} = 9$$

$$\frac{64+61}{2} = \text{Average of temp} = \frac{125}{2} = 62.5$$



Construct a curve interpolating data

Data Points

$x_0 \quad y_0 = f(x_0)$

$x_1 \quad y_1 = f(x_1)$

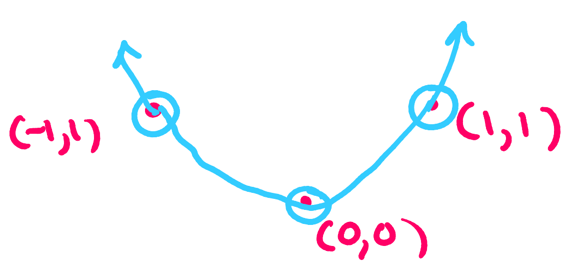
curve

$$p_1(x) = f(x_0) + \underbrace{f[x_0, x_1]}(x - x_0)$$

NEWTON'S D.D DIVIDED DIFFERENCE OF ORDER 1

$$\text{slope} = \frac{y_1 - y_0}{x_1 - x_0} = \frac{\Delta y}{\Delta x}$$

Recall



$x^2 = p_2(x)$

formula for poly interpolating 3 data points?

Last class: $p_2(x) = c\,x^2 + b x + a$

use $p_2(-1) = 1$

$p_2(0) = 0$ & $p_2(1) = 1$ to

figure out $a = 0 = b$, $c = 1$.

formula in lieu of the above approach is:

Newton's D.D formula:

x_i	y_i
x_0	y_0
x_1	y_1
x_2	y_2

$$p_2(x) = \text{Take } p_1(x) + \text{next term.}$$

$$= \underbrace{f(x_0) + f[x_0,x_1](x-x_0)}_{p_1(x)} +$$

$$f[x_0,x_1,x_2]\,(x-x_0)(x-x_1).$$

$$\frac{f[x_1,x_2] - f[x_0,x_1]}{x_2 - x_0}$$

Construct degree 2 polynomial interpolating the foll. data:

$\{(1,1), (2,2), (3,5)\}$ ($x_0\ f(x_0)$, $x_1\ f(x_1)$, $x_2\ f(x_2)$)

using Newton's D.D formula. Divided Difference

$$f[x_0,x_1] = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

x_i	$f(x_i)$	first order D.D $f[x_i, x_{i+1}]$	second order D.D $f[x_0,x_1,x_2]$
x_0 1	1		
x_1 2	2	$\frac{2-1}{2-1} = 1 = f[x_0,x_1]$	$\frac{f[x_1,x_2] - f[x_0,x_1]}{x_2 - x_0}$ (3, 1) $= 1 = f[x_0,x_1,x_2]$

x_1 (2) | 2 — | $2-1$ | $\dfrac{f[x_1,x_2] - f[x_0,x_1]}{x_2 - x_0} = 1 = f[x_0,x_1,x_2]$

x_2 (3) | 5 $\rightarrow \dfrac{5-2}{3-2} = \dfrac{3}{1} = f[x_1,x_2]$ $\qquad$ $\underset{3}{x_2} - \underset{1}{x_0}$

$$P_2(x) = \underline{f(x_0)} + \underline{f[x_0,x_1]}(x-\underset{1}{x_0}) + \underset{1}{\underline{f[x_0,x_1,x_2]}}(x-\underset{1}{x_0})(x-\underset{2}{x_1})$$

$$= 1 + (x-1) + (x-1)(x-2)$$

$$= x + x^2 - 3x + 2$$

$$P_2(x) = x^2 - 2x + 2$$

<u>check</u>: if $P_2(x)$ interpolates $(1,1), (2,2), (3,5)$

$P_2(1) = 1^2 - 2 + 2 = 1$ ✓

$P_2(2) = 4 - 2 * 2 + 2 = 2$ ✓

$P_2(3) = 3^2 - 6 + 2 = 9 - 6 + 2 = 5$ ✓

$P_2(x) = x^2 - 2x + 2$ is deg 2 poly interpolating $(1,1)$, $(2,2)$ & $(3,5)$.

<u>Advantage of Newton's D.D poly. formula</u>

can keep adding new data points.

$(1,1)$ & $(2,2)$ $\qquad$ $P_1(x) = f(x_0) + f[x_0,x_1](x-x_0)$

$(1,1)$ & $(2,2)$ and $(3,5)$ $\qquad$ $P_2(x) = P_1(x) + f[x_0,x_1,x_2](x-x_0)(x-x_1)$

and $(3,5)$ $P_2(x) = \ldots$

Use the Newton's D.D poly. formula to construct

$P_2(x)$ interpolating:

$(0,1)$, $(1,2)$ and $(2,3)$.

$x_0\ f(x_0)$ $\quad x_1\ f(x_1)$ $\quad x_2\ f(x_2)$

x_i	$f(x_i)$	$f[x_i, x_{i+1}]$	$f[x_0, x_1, x_2]$
0	1		
		$f[x_0,x_1] = \frac{2-1}{1-0} = 1$	
1	2		$f[x_0,x_1,x_2] = \frac{1-1}{\underset{x_2\ \ x_0}{2-0}} = 0$
		$f[x_1,x_2] = \frac{3-2}{2-1} = 1$	
2	3		

$$P_2(x) = f(x_0) + f[x_0,x_1](x-x_0) + f[x_0,x_1,x_2](x-x_0)(x-x_1)$$

$$= 1 + 1\ (x-0) + 0$$

$$P_2(x) = 1+x$$

$P_2(0) = 1$ ✓

$P_2(1) = 2$ ✓

$P_2(2) = 3$ ✓

Disadvantage: error representatn formula.

$G(x)$ = function for all data points

$G(x) \ldots P_2(x) \ldots$ has a poor* error representation

$G(x) =$ function ...

$G(x) - p_2(x)$ has a poor* error representation formula

↓ hard to calculate.

$|G(x) - p_2(x)| =$ Bad error formula.

Need another way of constructing the $p_2(x)$.

This other formula is called Lagrange interpolating poly. formula. $p_1^L(x)$

(x_0, y_0) $\quad (x_1, y_1) \longrightarrow p_1^L(x) = y_0 L_0(x) + y_1 L_1(x)$

$L_0(x)$: degree 1 poly vanishes at x_1 and is 1 at x_0

$L_1(x)$: degree 1 poly vanishes at x_0 and is 1 at x_1.

$$p_1^L(x_0) = y_0 L_0(x_0) + y_1 L_1(x_0)$$

$$= y_0 * 1 + y_1 * 0 = y_0$$

$$L_0(x) = \frac{(x_1 - x)}{(x_1 - x_0)} \qquad L_1(x) = \frac{x - x_0}{x_1 - x_0}$$

$$L_0(x_0) = 1, \; L_0(x_1) = 0 \qquad L_1(x_0) = 0 \quad L_1(x_1) = 1$$

3 interpolating points: (x_0, y_0) (x_1, y_1) & (x_2, y_2)

$$p_2(x) = y_0 L_0(x) + y_1 L_1(x) + y_2 L_2(x).$$

$$p_2(x) = y_0 L_0(x) + y_1 L_1(x) + y_2 L_2(x).$$

$L_0(x_0) = 1$

$L_0(x_1) = L_0(x_2) = 0$

$L_1(x_1) = 1$

$L_1(x_0) = L_1(x_2) = 0$

$L_2(x_2) = 1$

$L_2(x_0) = L_2(x_1) = 0$