Math 4329

Tuesday, March 10, 2020 4:29 PM

W.S 06 #2 $\quad f(x) = e^x - 2x - 1 \qquad 1 \le x \le 2.$

$\alpha = 1.2564.$

$$x_0 \to \left.\begin{aligned} x_1 &= g_1(x_0) \\ x_1 &= g_2(x_0) \\ x_1 &= g_3(x_0) \end{aligned}\right\} \to -1 < g_i'(\alpha) < 1 \qquad i = 1, 2, 3.$$

(b) Newton's Method (Easy)

$f(x) = e^x - 2x - 1 \qquad f'(x) = e^x - 2.$

$x_{n+1} = x_n + c f(x_n) \qquad c \ne 0$

AT LEAST Quad. convergence is guaranteed if $g'(\alpha) = 0$

The conv. is quad if $g''(\alpha) \ne 0$

Problem 1/WS06 : $g'(\alpha)$

$g(x) = 5 - (4+c)x + cx^5 \to g'(x) = 0 - (4+c) + 5cx^4$

$g'(1) = g'(\alpha) = -(4+c) + 5c$

$-1 < g'(\alpha) < 1, \ \alpha = 1$

Iter-3: $x_{n+1} = \ln(2x_n + 1) \to$ converges? Yes

$g(x) = \alpha \ln(x) - x$

$g'(x) = \frac{\alpha}{x} - 1 \qquad g'(\alpha) = 0$

$$\boxed{-1 < g'(\alpha) = 0 < 1}$$

$x1 = x0 - \frac{fx}{dfx}$

THE PRAGMATIC PROGRAMMER (useful prog. book)

$f(x) = x^3 - 3x^2 + 3x - 1, \quad \alpha = 1$

$f'(x) = 0$ when $x = 1$

repeated root at α.

↳ $f(x) = 0$ when $x = \alpha$

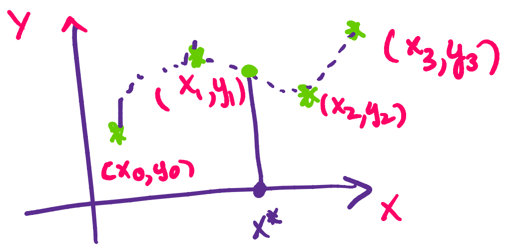
↳ $f(x)$ has a repeated root at α.

Inherent property of function : ill conditioned

$$x_{n+1} = x_n + c f(x_n) \quad , c \neq 0$$

---

chapter 4 on Interpolation :



Can we say something about $x^* = \dfrac{(x_1 + x_2)}{2}$

Curve fitting

X	Y
x_0	y_0
x_1	y_1
x_2	y_2

→ Can we say something about in between x values & y-values?

Use polynomial to interpolate $(x_0, y_0), \dots (x_n, y_n)$

n+1 distinct data points.

interpolate → pass a curve through these n+1 points.

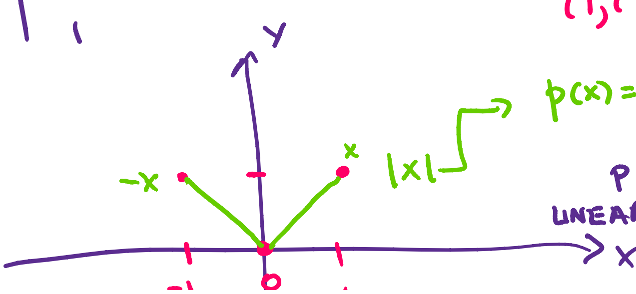
$$\boxed{p(x) = a + bx + cx^2}$$ should fit $(x_0, y_0), (x_1, y_1)$ & (x_2, y_2).

$$p(x_0) = y_0 \ , \quad p(x_1) = y_1 \ , \quad p(x_2) = y_2 .$$

Construction of an interpolating polynomial :

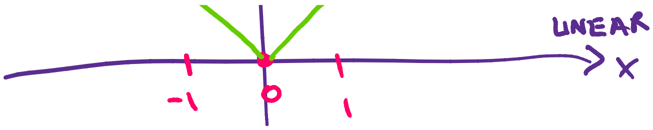
x	y
-1	1
0	0
1	1

Task: Construct a degree 2 poly $p(x)$ interpolating $(-1,1), (0,0)$ & $(1,1)$



$|x|$ → $p(x) = \begin{cases} -x & -1 \leq x < 0, \\ x & 0 \leq x < 1 \end{cases}$

PIECEWISE LINEAR POLYNOMIAL



LINEAR POLYNOMIAL

$\boxed{p(x) = a + bx + cx^2}$ a, b, c unknown & I

want to determine a, b & c so that:

$p(-1) = 1$, $p(0) = 0$ & $p(1) = 1$

$a + b*(-1) + c(-1)^2 = 1$, $a + b*0 + c*0^2 = 0$ & $\boxed{a+b+c=1}$

$\boxed{a - b + c = 1}$ $\boxed{a=0}$

$$\begin{aligned} a - b + c &= 1 \quad (1)\\ a &= 0 \quad (2)\\ a + b + c &= 1 \quad (3) \end{aligned}$$

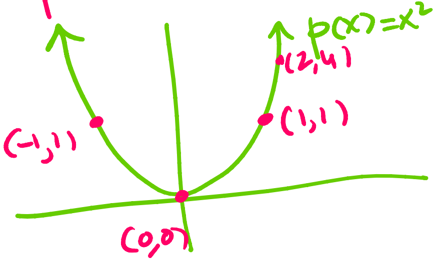
a=0 solve for b & c

use a=0 &

$(1) + (3) \Rightarrow 2c = 2 \Rightarrow c = 1$

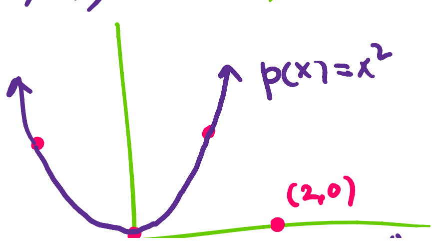
$(3) - (1) \Rightarrow 2b = 0 \Rightarrow b = 0$

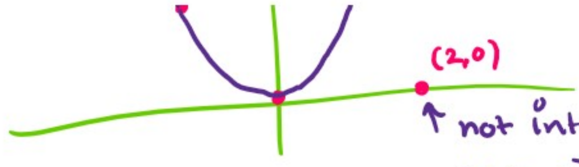
$p(x) = a + bx + cx^2 = 0 + 0x + x^2$
$= x^2$



Given another data point say (2,4) then $p(x) = x^2$ also would interpolate the additional data point (2,4).

(-1,1), (0,0), (1,1), (2,0)





↑ not interpolated by $p(x) = x^2$.

Given data:

$(-1,1)$, $(0,0)$, $(1,1)$, $(2,0)$ construct a degree 3 Interpolating polynomial $p_3(x)$.

x_i	y_i	$f[x_i,x_{i+1}] = \Delta y_i / \Delta x_i$	$f[x_i,x_{i+1},x_{i+2}]$	$f[x_0,x_1,x_2,x_3]$
-1	1	$\boxed{\frac{0-1}{0-(-1)}}$ $\frac{\Delta y_i}{\Delta x_i}$ -1		
0	0		$\frac{1-(-1)}{1-(-1)} = \boxed{1}$	$\boxed{\frac{-1-1}{2-(-1)} = -2/3}$
1	1	$\frac{1-0}{1-0}$ $\frac{\Delta y_i}{\Delta x_i}$ 1		
2	0	$\frac{0-1}{2-1}$ $\frac{\Delta y_i}{\Delta x_i}$ -1	$\frac{-1-1}{2-0} = \boxed{-1}$	

Poly $p_3(x) = \underset{1}{y_0} + \underset{-1}{f[x_0,x_1]}(x-x_0) + \boxed{f[x_0,x_1,x_2]}(x-x_0)(x-x_1)$
$+ f[x_0,x_1,x_2,x_3](x-x_0)(x-x_1)(x-x_2)$

$i=0$ $f[x_i,x_{i+1}] = (y_{i+1}-y_i)/x_{i+1}-x_i = \frac{0-1}{0-(-1)} = -1$

x_i	y_i y_0	$f[x_0,x_1]$	$f[x_0,x_1,x_2]$	$f[x_0,x_1,x_2,x_3]$
-1	1	-1	1	$-2/3$
0	0			
		1		
1	1		-1	
2	0	-1		

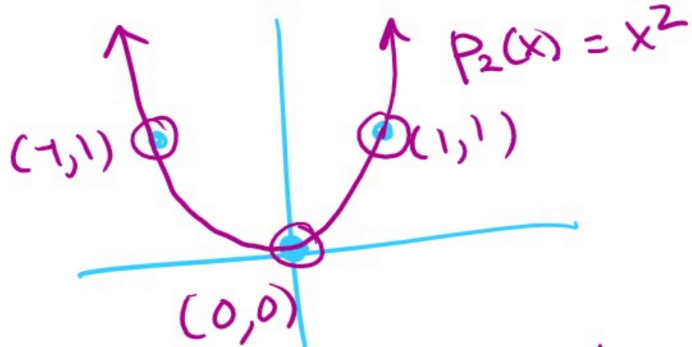
$p_3(x) = y_0 + f[x_0,x_1](x-x_0) + f[x_0,x_1,x_2](x-x_0)(x-x_1) + (-2/3)(x-x_0)(x-x_1)(x-x_2)$

$\boxed{x_0 = -1}$ $x_1 = 0$ $x_2 = 1$

$= 1 + (-1)(x-(-1)) + 1 * \underset{(x-x_0)}{\overset{(x-x_0)}{(x-(-1))}}(x-0) + (-2/3)(x-(-1))(x-0)(x-1).$

formula for deg 3 poly interpolating

$(-1,1)$, $(0,0)$, $(1,1)$ & $(2,0)$
$(x_0\ y_0)$ $(x_1\ y_1)$ (x_2, y_2) $(x_3\ y_3)$



<u>Note</u>: $p_2(x) = x^2$ is obtained if we apply the above method.

x	y	$f[x_i, x_{i+1}]$	$f[x_0, x_1, x_2]$
$x_0 = -1$	$\boxed{1}$ y_0	$f(x_0, x_1]$: $\dfrac{0-1}{0-(-1)} = \boxed{-1}$	$\dfrac{1-(-1)}{x_2 - x_0} = \dfrac{1+1}{1+1} = 1$
$x_1 = 0$	0		$x_2 - x_0 = 1-(-1)$
$x_2 = 1$	1	$\dfrac{1-0}{1-0} = 1$	

$$p_2(x) = y_0 + f[x_0, x_1]\,(x - x_0) + f[x_0, x_1, x_2](x - x_0)(x - x_1)$$

$$= 1 + (-1)(x-(-1)) + 1 * (x-(-1))(x-0)$$

$$= 1 - (x+1) + (x+1)x$$

$$= \cancel{1} - x - \cancel{1} + x^2 + x$$

x^2 same as before when $p_2(x) = a + bx + cx^2$!

$= x^2$ same as before when $p_2(x) = a + bx + cx^2$!