Math 4329

Thursday, February 27, 2020 3:41 PM

f(x) = −x 4 + 3x 2 + 2

Last class: applications of Newton's Method

Replace complicated function & transform to Root finding problem.

$\sqrt{3} \to ?$ solve for $x = +\sqrt{3}$?

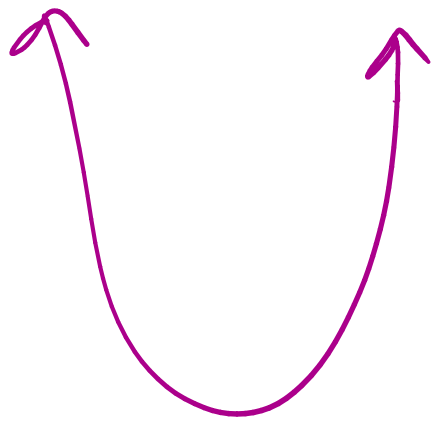
$x = +\sqrt{3} \longrightarrow x^2 = (+\sqrt{3})^2 = 3$

$$\boxed{x^2 = 3}$$

$x^2 - 3 = 0$

$-\sqrt{3}$ $+\sqrt{3}$

$f(x) = x^2 - 3$ $x_0 = 1.7$



Second Application: DIVISION

Replace the operation $b > 0$

$1/b = ?$

find x such that $x = 1/b$

* No Division operation involved!

~~$x = \sqrt{3} \to x^2 = 3$~~

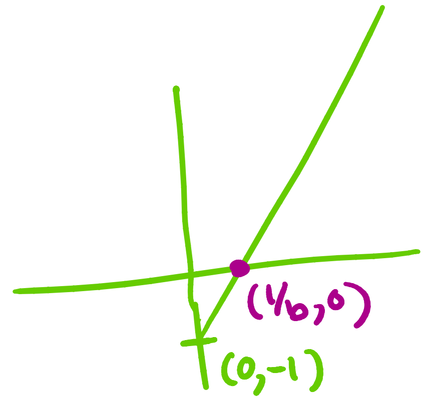
$x = 1/b$

$x = 1/b$

$b * x = 1$

$b x = 1$

$f(x) := bx - 1 = 0$



$$\boxed{f(x) = bx - 1}$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \qquad n = 0, 1, \ldots$$

$$= x_n - \frac{(bx_n - 1)}{b}$$

$$\boxed{x_{n+1} = x_n - \frac{bx_n}{b} + 1/b = 1/b}$$

Does not serve the purpose!

find x: $x = 1/b \rightarrow b \to 0$

$$\boxed{f(x) = bx - 1} \rightarrow f(x) = 0$$

$x_{n+1} = 1/b$

find a form for $f(x)$ so that there is no division involved in

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$\boxed{x = 1/b} \rightarrow bx = 1$$

$$\rightarrow \frac{b\cancel{x}}{\cancel{x}} = \frac{1}{x}$$

$$\boxed{b = 1/x}$$

$$f(x) = b - 1/x \qquad f'(x) = 0 - \left(-1/x^2\right) = 1/x^2$$

$$f'(x) = 1/x^2$$

$$\frac{1}{f'(x)} = x^2$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \qquad f(x) = b - 1/x$$

$$x_{n+1} = x_n - \frac{(b - 1/x_n)}{1/x_n^2} = x_n - x_n^2(b - 1/x_n)$$

$$= x_n - bx_n^2 + x_n^2/x_n$$

$$= x_n - bx_n^2 + x_n$$

$$= 2x_n - bx_n^2$$

$$\boxed{x_{n+1} = 2x_n - bx_n^2} \text{ solves } \boxed{x = 1/b}$$

<u>Remark</u>: $f(x) := x^3 - 3x^2 + 3x - 1$

apply Newton's method with $\varepsilon = 10^{-6}$.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$ careful about subtraction

one way to avoid any possible loss-of-significance errors due to subtraction is to simplify the formula $x_{n+1} = x_n - f(x_n)/f'(x_n)$

<u>useful formula</u>:

$$a^2 - b^2 = (a-b)(a+b)$$

$$a^3 - b^3 = (a-b)(a^2 + b^2 + ab)$$

<u>example</u>: Simply the applicatn of Newton's method to $f(x) = x^2 - 3$.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \frac{(x_n^2 - 3)}{3x_n}$$

$$= \frac{3x_n x_n}{3x_n} - \frac{(x_n^2 - 3)}{3x_n}$$

$$= \frac{3x_n^2 - (x_n^2 - 3)}{3x_n} = \frac{2x_n^2 + 3}{3x_n}$$

Remark: 3.2 #3(a) special case with $a=3$.

Solve $x^2-a=0$ using Newton's method &

$$x_{n+1} = \frac{1}{2}\left(x_n + a/x_n\right)$$

$$f(x) = x^2 - a$$
$$f'(x) = 2x$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$f(x) = x^3 - 3x^2 + 3x - 1 \rightarrow 3x^2 - 6x + 3 = f'(x)$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \frac{(x_n^3 - 3x_n^2 + 3x_n - 1)}{3x_n^2 - 6x_n + 3}$$

$$= \frac{(3x_n^2 - 6x_n + 3)}{(3x_n^2 - 6x_n + 3)} x_n - \frac{(x_n^3 - 3x_n^2 + 3x_n - 1)}{3x_n^2 - 6x_n + 3}$$

$$= \frac{(3x_n^2 - 6x_n + 3)x_n - (x_n^3 - 3x_n^2 + 3x_n - 1)}{3x_n^2 - 6x_n + 3}$$

$$= \frac{(3x_n^2 - 6x_n + 3)x_n - (x_n^3 - 3x_n^2 + 3x_n - 1)}{3x_n^2 - 6x_n + 3}$$

$$= \frac{3x_n^3 - 6x_n^2 + \cancel{3x_n} - x_n^3 + 3x_n^2 - \cancel{3x_n} + 1}{3x_n^2 - 6x_n + 3}$$

$$= \frac{2x_n^3 - 3x_n^2 + 1}{3x_n^2 - 6x_n + 3}$$ simplified expression!

Secant Method:

$$f(x) = x^2 - x \qquad x_0 = 1/2$$

apply Newton's method

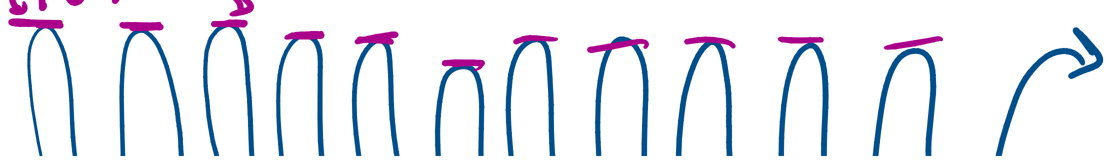
$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

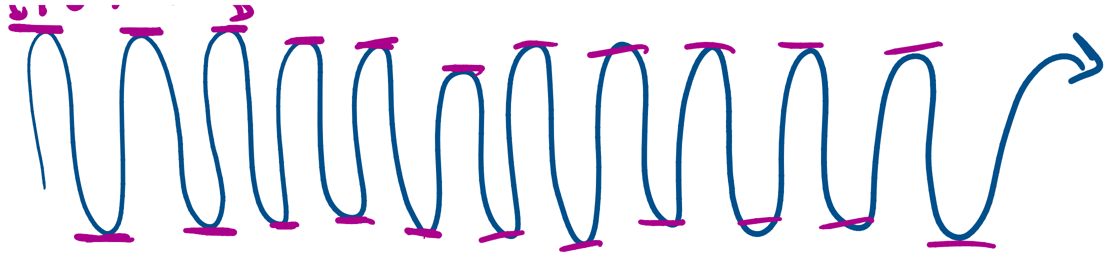
$$= x_0 - \frac{(x_0^2 - x_0)}{2x_0 - 1}$$

$$= 0.5 - \frac{(0.25 - 0.5)}{2 * 0.5 - 1} = 0.5 - \frac{(-0.25)}{0}$$

Derivative $f'(x_0)$ d.n.e!

$f'(x) = 0$





$$f'(x_1) \approx \frac{f(x_1)-f(x_0)}{x_1-x_0}$$

Initial Guess x_0 & x_1

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

No derivatives involved replace derivative by

$$\frac{f(x_1)-f(x_0)}{x_1-x_0}$$