Math 4329

Thursday, February 20, 2020 4:36 PM

Bisection Method : $x^6 - x - 1 = 0$

find a & b such that the interval $[a, b]$

contains a root to the equation:

$$x^6 - x - 1 = 0$$

$f(1) = 1^6 - 1 - 1 = -1$ $\quad f(2) = 2^6 - 2 - 1 > 0$

for $\varepsilon = 10^{-3}$ we want to determine the number of midpoints c_n needed to obtain

$$|\alpha - c_n| < \varepsilon = 10^{-3}$$

Note: α is a root of $x^6 - x - 1 = 0$.

WE WANT TO FIND n without calculating c_ns.

error = | True Value - Approx. value |

α $\qquad$ c_n

Problem: We don't know α.

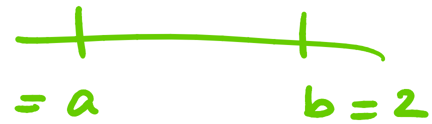
$$|\alpha - c_n| \leq \frac{1}{2^n}(b-a)$$

Figure out n :

$$\frac{1}{2^n}(b-a) < \varepsilon = 10^{-3}$$

$$\frac{1}{2^n}(b-a) < \varepsilon = 10^{-3}$$

$$\Rightarrow |\alpha - c_n| \leq \frac{1}{2^n}(b-a) < \varepsilon$$



Given:

$\varepsilon = 10^{-3}$ $a = 1$, $b = 2$

find n such that

$$\frac{2-1}{2^n} < 10^{-3} \quad \text{holds.}$$

$$\frac{1}{2^n} < 10^{-3}$$

Try values of n which satisfy $\frac{1}{2^n} < 10^{-3}$.

Another way (more math. approach involves log)

$$\frac{b-a}{2^n} < \varepsilon$$

$$\log\left(\frac{b-a}{2^n}\right) < \log(\varepsilon)$$

use

$$\log(A/B) = \log(A) - \log B$$

$$\log(c^n) = n \log c$$

$$\log(b-a) - \log(2^n) < \log \varepsilon$$

$$\log(b-a) - n\log 2 < \log \varepsilon + n \log 2$$

$$\log(b-a) - n\log 2 < \log\varepsilon + n\log 2$$
$$-\log\varepsilon \quad + n\log 2 \qquad -\log\varepsilon$$

$$\log(b-a) - \log\varepsilon < n\log 2$$

$$\frac{\log(b-a) - \log\varepsilon}{\log 2} < n$$

$$\boxed{\frac{\log\left((b-a)/\varepsilon\right)}{\log 2} < n}$$

Page 74, eq^n (3.9)

find n satisfying

$a=1, b=2, \varepsilon = 10^{-3}$

$$\frac{\log\left((2-1)/10^{-3}\right)}{\log 2} < n$$

$$\frac{\log(10^3)}{\log 2} < n$$

$9.96 < n \Rightarrow n \geq 10$

... to compute 10 midpoints c_n

We need to compute 10 midpoints c_n

So that $|\alpha - c_n| < \frac{1}{2^n}(b-a) < 10^{-3}$.

exercise: $\boxed{x \log x = \cos x}$ $\varepsilon = 10^{-3}$.

using MATLAB, write a program bisect.m that

input: a0 → "a"

b0 → "b"

ep → "$\varepsilon = 10^{-3}$"

max_iterate → max. # of midpoints to be calculated.

Look at course website.

Program → MATLAB SCRIPT → Lines of commands

↳ M-FUNCTION

m-function : filename.m

function output = filename (a0, b0, ε, max_iterat)

↑ ↑ ↑ ↑ input arguments.

$\boxed{\text{bisect.m}}$

- bisect function

↳ bisect function

need to define

$$f(x) = x^6 - x - 1$$

Remark: % used for comments.

$f(a)\,f(b) < 0 \longrightarrow$ If $f(a)\,f(b) > 0$ then stop!

$$\left\{ \begin{array}{ll} B1 & c = \frac{a+b}{2} \\ B2 & b - c \leq \varepsilon \\ B3 & \end{array} \right\}$$

If $a > b \to$ stop!

exam 01 solutions

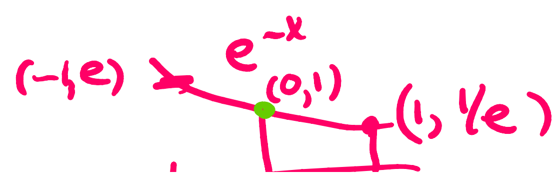
① $\log 0 \quad \log(-\pi/2)$ log domain positive numbers.

② $$e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \cdots R_n(x)$$

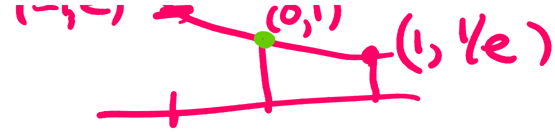
$$\left| e^{-x} - p_n(x) \right| \leq e^{-c} \frac{x^{n+1}}{(n+1)!} \qquad -1 \leq x \leq 1$$

0

e^0



e^0



③ $\frac{\sqrt{9+x}-3}{x}$

$\frac{\sqrt{9+x}+3}{\sqrt{9+x}+3}$

$\frac{e^x - e^{-x}}{2x}$

$\frac{e^x - (1 - x + x^2/2!)}{2x}$

⑤ $1/e$ using $p_3(x)$.

$$e^{-x} \approx 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!}$$

Replace x with -1

$$e^{-1} \approx 1 - 1 + 1/2 - (-1/3!)$$

$\frac{\text{abs error}}{\text{true value}}$

e^{-1}