Math-4329

Thursday, January 30, 2020 4:33 PM

$45.12 \rightarrow \cdots a_4 a_3 a_2 a_1 a_0 . b_1 b_2 b_3 \cdots$

$+ a_0 2^0 + a_1 2^1 + \cdots$

$b_1 2^{-1} + b_2 2^{-2} + \cdots$

b_i & a_i are 0 or 1

$2 \overline{)\, 45\ (22}$

44

$1 \rightarrow$ last coeff

$2 \overline{)\, 22\ (11}$

22

$0 \rightarrow$ second last coeff

$2 \overline{)\, 11\ (5}$

10

$1 \rightarrow$ 3rd last coeff

$2 \overline{)\, 5\ (2}$

4

$1 \rightarrow 4^{th}$ last coeff

$2 \overline{)\, 2\ (1}$

2

$0 \rightarrow 5^{th}$ last coeff

$2 \overline{)\, 1\ (0}$

0

$1 \rightarrow 6^{th}$ last coeff

45

1	0	1	1	0	1
2^5	2^4	2^3	2^2	2^1	2^0
last coeff	2nd last	3rd last	4^{th} last	5^{th} last	6^{th} last

Binary format

0.12 → Binary format

$0.12 * 2 = 0.24 \rightarrow b_1$

$0.24 * 2 = 0.48 \rightarrow b_2$

$0.48 * 2 = 0.96 \ldots$

$0.24 * 2 = 0.48 \rightarrow b_2$

$0.48 * 2 = 0.96 \rightarrow b_3$

$0.96 * 2 = 1.92 \rightarrow b_4$

When we hit integer part we subtract 1.

$0.92 * 2 = 1.84 \rightarrow b_5$

$0.84 * 2 = 1.68 \rightarrow b_6$

$0.68 * 2 = 1.36 \rightarrow b_7$

$0.36 * 2 = 0.72 \rightarrow b_8$

REVISIT THIS SHORTY! (See end of notes!!) $\cdot b_1 b_2 b_3 \ldots$

$0.125 \longrightarrow$ Binary format $2^{-3} = 1/8 = 0.125$

$0.125 = 1/8 = 2^{-3} = (.001)_2$ (2^{-1} 2^{-2} 2^{-3})

$0.125 * 2 = 0.25 \rightarrow b_1$ coefficient of 2^{-1}

$0.25 * 2 = 0.5 \rightarrow b_2$ " of 2^{-2}

$0.5 * 2 = 1.00 \rightarrow b_3 = 1$ coeff of 2^{-3}

$0.00 * 2 = 0 \rightarrow b_4$

$0.00 * 2 = 0 \rightarrow b_5$.

$0.125 \longrightarrow (0.001)_2$

example: $0.1 \longrightarrow$ Binary format?

$0.1 \times 2 = 0.2 = b_1 = 0$

$0.2 \times 2 = 0.4 \quad b_2 = 0$

$b_3 = 0$

$0.2 \times 2 = 0.4$ $b_2 = 0$

$0.4 \times 2 = 0.8$ $b_3 = 0$

$0.8 \times 2 = 1.6$ $b_4 = 1$ } $= b_8$

$0.6 \times 2 = 1.2$ $b_5 = 1$ } $= b_9$

$0.2 \times 2 = 0.4$ $b_6 = 0$ } $= b_{10}$

$b_7 = 0$ } $= b_{11}$

$0.1 \longrightarrow (0\ \underline{0011}\ \underline{0011}\ 0011\ldots)_2$

↓ gets repeated

Hexadecimal format :

$0.1 \longrightarrow \underset{b_1}{0} * 2^{-1} + \underset{b_2}{0} * 2^{-2} + \underset{b_3}{0} * 2^{-3} + \underset{b_4}{1} * 2^{-4} + \underset{b_5}{1} * 2^{-5}$

$+ \underset{b_6}{0} * 2^{-6} + \underset{b_7}{0} * 2^{-7} + 1 * 2^{-8} + 1 * 2^{-9}$

$+ \ldots$

Base = 2 coefficients are integers greater than equal to 2 but smaller than 2.

$\{0, 1\}$

Base = 10 $\longrightarrow$ coefficients are $\{0, 1, \ldots 9\}$

$95 = 90 + 5$

$= \underset{a_1}{9} \times 10^1 + \underset{a_0}{5} * 10^0$

Base 16 $\longrightarrow$ coefficients lie in $\{0, 1, \ldots 15\}$

0 10 → A

1 11 → B

2 12 → C

⋮ 13 → D

9 14 → E

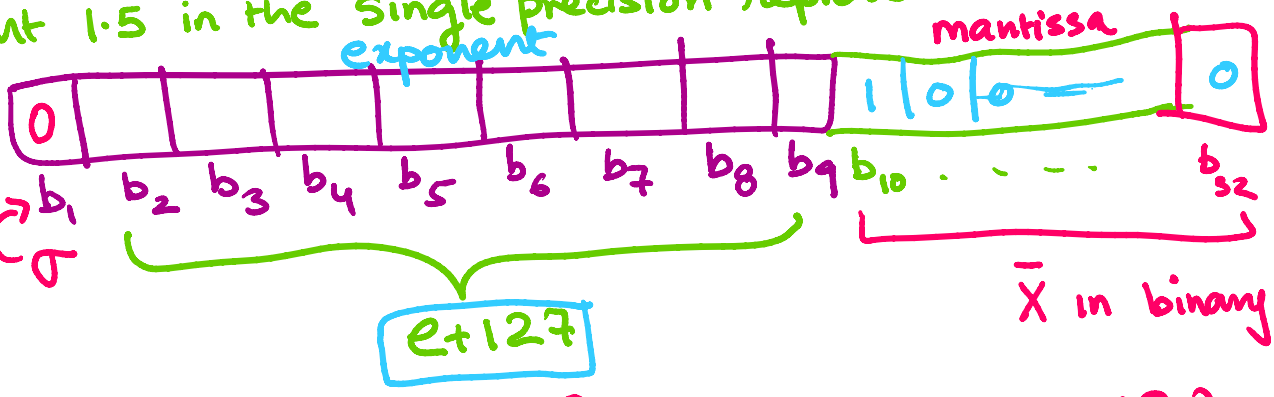
15 → F

Representation of a number in hexa is

TEFF

Representation of a number in hexa is related to its IEEE double precision representation

Represent 1.5 in the single precision representation.



σ : $(-1)^0$

$e+127$

$\bar{X}$ in binary

$$X = \sigma \cdot \bar{X} \cdot 2^e \qquad -126 \le e \le 127$$

(1.5) → convert in binary format

↳ $(0001.100)_2$ $\bar{x} \to 1.1$

↳ Donot store 1

$1.5 \to 2^0 + 2^{-1}$

$X = 1 * 1.1 * 2^0$ $e = 0$

only 0.1 → gets stored

$e = 0 \to e+127$

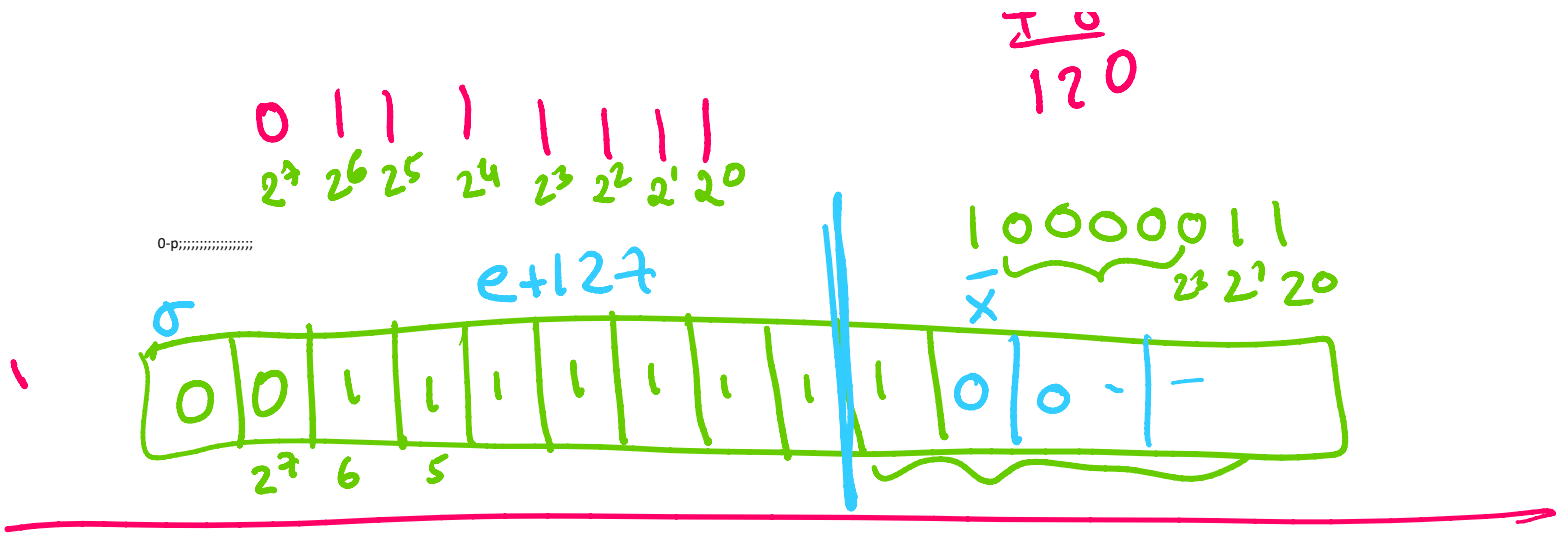
store 127 → Binary format ?

Priscilla 2^8 6^4 3^2 16 8 4 2 1

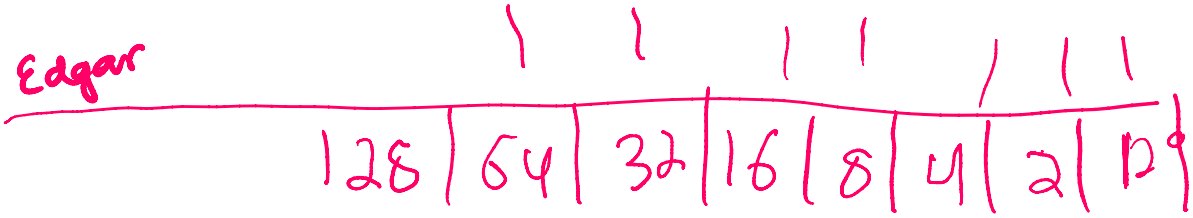
0 1 1 1 1 1 1 1

1

96
+14
112
+8
120

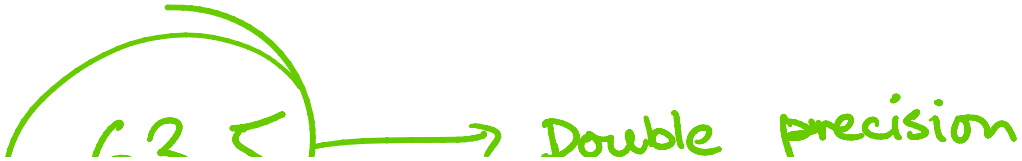


Edgar

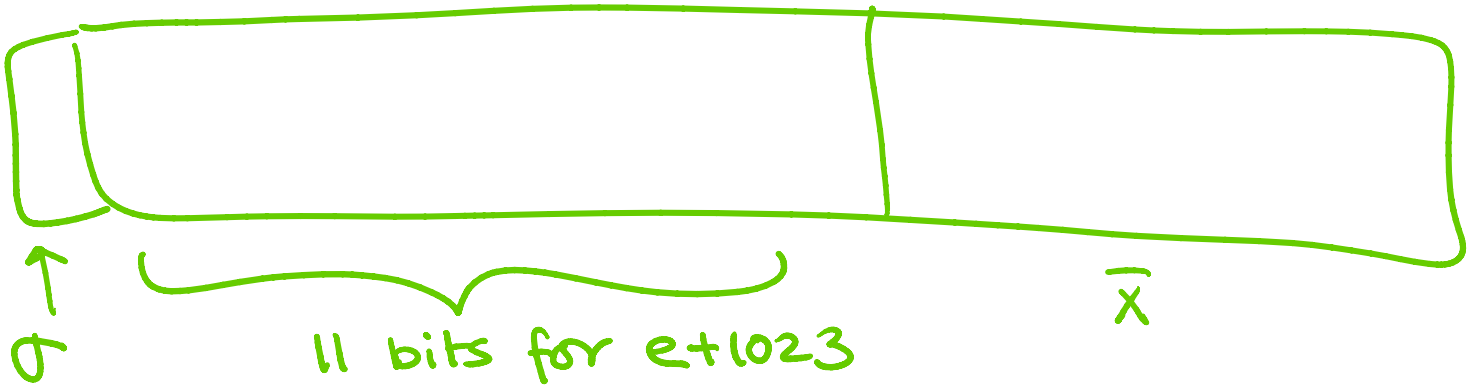


Afua

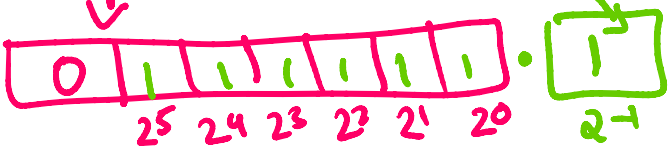
2	127	R
2	63	1
2	31	1
2	15	1
2	7	1
2	3	1
	1	1



63.5 → Double precision

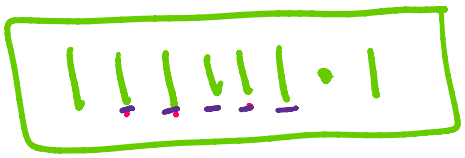


$63.5 = 63 + 2^{-1}$



$2^6 - 1 = 2^0 + \cdots 2^5$

represent in $\sigma\ \bar{x}\ 2^e$



$e = 5$

$e + 1023 = 1028$

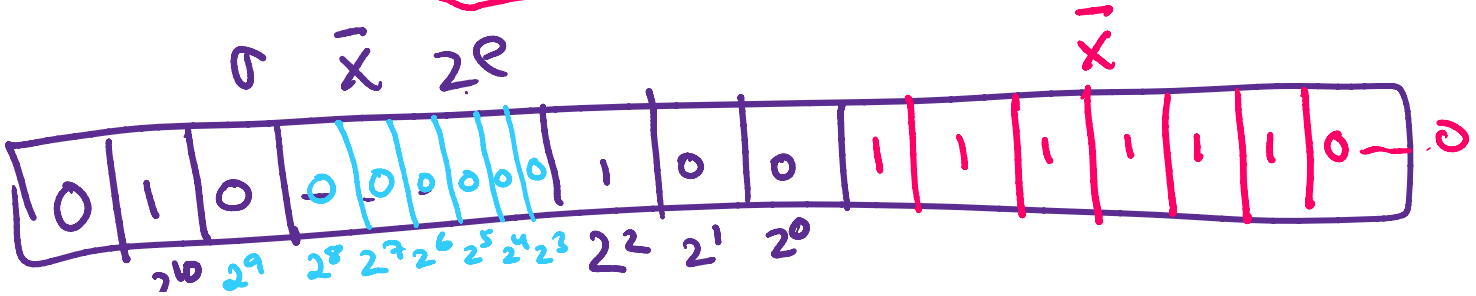
$= 2^{10} + 2^2$

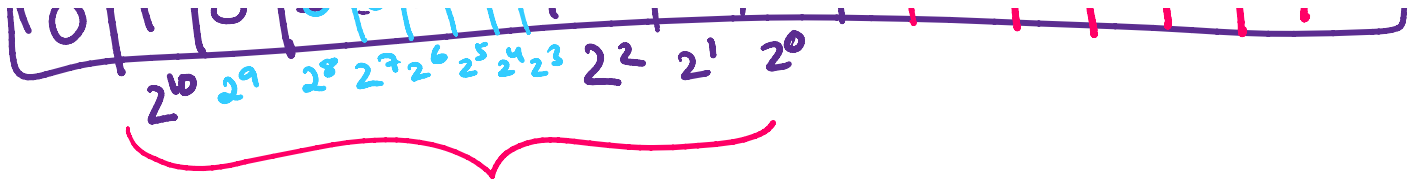
1.111111

$\bar{x}$

$1 * (1.111111)\ 2^5$

$\sigma\ \bar{x}\ 2^e$





e+1023

final answer

EXTRA Question: Find the number in decimal which has the following binary representation

$(111\,000\,000.11)_2$

$(\underset{2^8}{1}\ \underset{2^7}{1}\ \underset{2^6}{1}\ \underset{2^5\,2^4\,2^3}{000}\ \underset{2^2\,2^1\,2^0}{000} \cdot \underset{2^{-1}\,2^{-2}}{11})_2$

Answer: $2^8 + 2^7 + 2^6 + 2^{-1} + 2^{-2}$

$(256 + 128 + 64)$ $\quad$ $0.5 + 0.25$

448.75 is the number having the binary Representation $(111\,000\,000.11)_2$

Find the hexadecimal representation of 730

$730 = 2 * 16^2 + 218$

208 $\quad$ 10

$= 2 * 16^2 + (13) * 16^1 + (10) * 16^0$

D $\quad$ A

A → 10
B → 11
C → 12
D → 13

Answer: $(730)_{10} = (2DA)_{16}$

Answer: $(730)_{10} = (2DA)_{16}$

Back to the binary format of 0.12

$0.12 * 2$	$= 0.24$	$b_1 = 0$
$0.24 * 2$	$= 0.48$	$b_2 = 0$
$0.48 * 2$	$= 0.96$	$b_3 = 0$
$0.96 * 2$	$= 1.92$	$b_4 = 1$
$0.92 * 2$	$= 1.84$	$b_5 = 1$
$0.84 * 2$	$= 1.68$	$b_6 = 1$
$0.68 * 2$	$= 1.32$	$b_7 = 1$
$0.32 * 2$	$= 0.64$	$b_8 = 0$
$0.64 * 2$	$= 1.28$	$b_9 = 1$
$0.28 * 2$	$= 0.56$	$b_{10} = 0$
$0.56 * 2$	$= 1.12$	$b_{11} = 1$
$0.12 * 2$	$= 0.24$	$b_{12} = 0$

0 0 0 1 1 1 1 0 1 0 1 0

Answer

$0.12_{10} = 0.\overline{0001111010}$

Repeating & non terminating!