Numerical Analysis: Applications

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Numerical Analysis Revisited

Definition

Numerical analysis is a branch of mathematics that solves continuous problems using numeric approximation.
It involves designing methods that give approximate but accurate numeric solutions, which is useful in cases where the exact solution is impossible or prohibitively expensive to calculate.
Numerical analysis also involves characterizing the convergence, accuracy, stability, and computational complexity of these methods.

Powerful Applications of Numerical Analysis

- Solving a system of nonlinear equations
- Prediction of growth trends based on past data (application of interpolation)
- Solving Differential Equations
- Develop large scale software to solve Mathematical problems such as LaPack, Trilinos.
- Root finding Problems in n dimensions arises when we try to optimize a function $f(x, y, z)$ and to find the extrema we solve a system of equations for (x^*, y^*, z^*) satisfying

$$\partial_x f(x^*, y^*, z^*) = 0$$
$$\partial_y f(x^*, y^*, z^*) = 0$$
$$\partial_z f(x^*, y^*, z^*) = 0$$

Interpolation Application

Example

Suppose our task is to determine the net income for year 2019 based on the net incomes given below

Year	Net Income
2016	48.3 million
2017	90.4 million
2018	249.9 million

Numerical Differentiation

Example

Suppose that in the winter the daytime temperature in a certain office is maintained at 70 degrees F. The heating is shut off at 10 pm and turned on again at 6 am. On a certain day the temperature inside the building at 2 am was found to be 65 degrees F. The outside temperature was 50 degrees at 10 pm and dropped to 40 degrees F by 6 am.
What is the temperature in the building when the heat was turned on at 6 am?

Experimental data: Experiments show that the time rate of change of temperature T of a body B is proportional to the difference between T and the temperature of the surrounding medium.(Newton's Law of Cooling)

$$\frac{dT}{dt} = k(T - T_A)$$

Numeical Solutions to IVP

Since the computers cannot store or understand continuous time, we need to discretize the time interval $[0, 8]$ into say $N + 1$ "time screenshots" where we choose to solve for $T(t)$. We set

$$\Delta t = \frac{8 - 0}{N}$$

and let $t_0 = 0,\ t_1 = \Delta t,\ t_2 = 2\Delta t, \cdots t_N = 8.$

Time Screenshot	Approximation
$0 = t_0$	$T_0 = T(0)$
$\Delta t = t_1$	$T_1 = T(\Delta t)$
$2\Delta t = t_2$	$T_2 = T(2\Delta t)$
$3\Delta t = t_3$	$T_3 = T(3\Delta t)$
$\vdots$	$\vdots$
$N\Delta t = 8$	$T_N = T(8)$

x_i	y_i
$0 = t_0$	$T_0 = T(0)$
$\Delta t = t_1$	$T_1 = T(\Delta t)$
$2\Delta t = t_2$	$T_2 = T(2\Delta t)$
$3\Delta t = t_3$	$T_3 = T(3\Delta t)$
$\vdots$	$\vdots$
$N\Delta t = 8$	$T_N = T(8)$

Solve for temperature at $i = 1, 2, \cdots N$

$$\frac{T(t_{i+1}) - T(t_i)}{t_{i+1} - t_i} \approx \frac{dT}{dt} = k(T(t_i) - T_A).$$