Section 9.8

Definition of Power Series: If x is a variable, then an infinite series of the form

$$\sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots + a_n x^n + \cdots$$

is called a **power series**. More generally, an infinite series of the form

$$\sum_{n=0}^{\infty} a_n x^n = a_0 + a_1(x-c) + a_2(x-c)^2 + a_3(x-c)^3 + \cdots + a_n(x-c)^n + \cdots$$

is called a **power series centered at c,** where c is a constant.

Convergence of a Power Series: For a power series centered at c, precisely one of the following is true.

1. The series converges only at $x = c$.
2. There exists a real number $R > 0$ such that the series converges absolutely for $|x - c| < R$, and diverges for $|x - c| > R$.
3. The series converges absolutely for all x.

The number R is the **radius of convergence** of the power series. If the series converges only at c, the radius of convergence is $R = 0$, and if the series converges for all x, the radius of convergence is $R = \infty$. The set of all values of x for which the power series converges is the **interval of convergence** of the power series.

1) Find the radius of convergence for the power series.

$$\sum_{n=0}^{\infty} \frac{(-1)^n (x-1)^{2n}}{(2n)!}$$

2) Find the radius of convergence for the power series.

$$\sum_{n=1}^{\infty} \frac{x^n}{n^3}$$

3) Find the radius of convergence for the power series.

$$\sum_{n=0}^{\infty} \frac{(-1)^{n+1}(x-1)^{2n}}{4^n}$$

4) Find the interval of convergence for the power series.

$$\sum_{n=1}^{\infty} \frac{4^n(x-2)^n}{5n}$$

5) Find the interval of convergence for the power series.

$$\sum_{n=1}^{\infty} \frac{n^3(x-3)^n}{2^{n+1}}$$

6) Let

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}.$$

Write out the first six terms of this power series, and show that $f''(x) = -f(x)$. Which function might this power series represent?

Homework for 9.8: #7, 15, 19, 21, 29, 30, 43, 47, 59