Section 9.3

The Integral Test: If f is positive, continuous, and decreasing for $x \geq 1$ and $a_n = f(n)$, then

$$\sum_{n=1}^{\infty} a_n \quad \text{and} \quad \int_1^{\infty} f(x)dx$$

either both converge or both diverge.

Convergence of p-Series: The p-series

$$\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \frac{1}{4^p} + \cdots$$

1. converges if $p > 1$, and
2. diverges if $0 < p \leq 1$.

1) Apply the Integral Test to the series

$$\sum_{n=1}^{\infty} \frac{2n}{n^2+1}.$$

2) Apply the Integral Test to the series

$$\sum_{n=0}^{\infty} ne^{-n^2}$$

3) Determine whether the following series converge or diverge.

a) $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n}}$

b) $\sum_{n=1}^{\infty} \frac{2}{\sqrt{n^3}}$

Homework for 9.3: #3, 9, 16, 35, 36

Section 9.4

Direct Comparison Test: Let $0 < a_n \leq b_n$ for all n.

1. If $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.
2. If $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} b_n$ diverges.

Limit Comparison Test: Suppose that $a_n > 0, b_n > 0$, and

$$\lim_{n\to\infty} \left(\frac{a_n}{b_n}\right) = L$$

where L is *finite and positive*. Then the two series $\sum a_n$ and $\sum b_n$ either both converge or both diverge.

1) Determine the convergence or divergence of the following series.

a) $\sum_{n=1}^{\infty} \frac{1}{n^2+3}$

b) $\sum_{n=1}^{\infty} \frac{n^2+1}{n^3-2n}$

2) Use the limit comparison test to determine if the following series converge or diverge.

a) $\sum_{n=2}^{\infty} \frac{n}{n^3-1}$

b) $\sum_{n=1}^{\infty} \frac{n^2}{n^3+3}$

c) $\sum_{n=2}^{\infty} \frac{1}{\ln n}$

d) $\sum_{n=1}^{\infty} \frac{6^n}{7^n-n}$

Homework for 9.4: #3, 8, 9, 13, 17, 19