Section 9.10

The Form of a Convergent Power Series: If f is represented by a power series $f(x) = \sum a_n(x-c)^n$ for all x in an open interval containing c, then

$$a_n = \frac{f^{(n)}(c)}{n!}$$

and

$$f(x) = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \cdots + \frac{f^{(n)}(c)}{n!}(x-c)^n + \cdots.$$

Definition of Taylor and Maclaurin Series: If a function f has derivatives of all orders at $x = c$ then the series

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(c)}{n!}(x-c)^n = f(c) + f'(c)(x-c) + \cdots + \frac{f^{(n)}(c)}{n!}(x-c)^n + \cdots$$

is called the **Taylor series for $f(x)$ at c.** Moreover, if $c = 0$, then the series is the **Maclaurin series for f.**

1) Use the definition of Taylor series to find the Taylor series, centered at c, for the function.

   a) $f(x) = \frac{1}{1-x}$, $\quad c = 2$

   b) $f(x) = e^{-4x}$, $\quad c = 0$

2) Use the binomial series to find the Maclaurin series for the function $f(x) = \frac{1}{\sqrt{1-x^2}}$.

3) Find the Maclaurin series for the function. Use the table of power series for elementary functions on p. 670.

a) $f(x) = \ln(1 + x^2)$

b) $f(x) = \sin(\pi x)$

c) $f(x) = \cos^2 x$

4) Find the Maclaurin series for the function $f(x) = x \cos x$.

5) Find the first four nonzero terms of the Maclaurin series for the function by multiplying or dividing the appropriate power series.

a) $f(x) = e^x \cos x$

b) $f(x) = \frac{e^x}{1+x}$

6) Find a Maclaurin series for $f(x) = \int_0^x \sqrt{1 + t^3}\, dt$.

Homework for 9.10: #3, 9, 21, 35, 38, 47, 51