Section 9.1

Definition: A **sequence** is defined as a function whose domain is the set of positive integers.

The Limit of a Sequence: Let L be a real number. Let f be a function of a real variable such that

$$\lim_{x\to\infty} f(x) = L.$$

If $\{a_n\}$ is a sequence such that $f(n) = a_n$ for every positive integer n. Then

$$\lim_{n\to\infty} a_n = L.$$

Squeeze Theorem for Sequences: If

$$\lim_{n\to\infty} a_n = L = \lim_{n\to\infty} b_n$$

and there exists an integer N such that $a_n \leq c_n \leq b_n$ for all $n > N$, then

$$\lim_{n\to\infty} c_n = L.$$

Absolute Value Theorem: For the sequence $\{a_n\}$, if

$$\lim_{n\to\infty} |a_n| = 0 \quad \text{then} \quad \lim_{n\to\infty} a_n = 0.$$

Definition of Monotonic Sequence: A sequence $\{a_n\}$ is **monotonic** if its terms are nondecreasing

$$a_1 \leq a_2 \leq a_3 \leq \cdots \leq a_n \leq \cdots$$

or if its terms are nonincreasing

$$a_1 \geq a_2 \geq a_3 \geq \cdots \geq a_n \geq \cdots.$$

Definition of Bounded Sequence:

1. A sequence $\{a_n\}$ is **bounded above** if there is a real number M such that $a_n \leq M$ for all n. The number M is called an **upper bound** of the sequence.
2. A sequence $\{a_n\}$ is **bounded below** if there is a real number N such that $N \leq a_n$ for all n. The number N is called an **lower bound** of the sequence.
3. A sequence $\{a_n\}$ is **bounded** if it is bounded above and bounded below.

Bounded Montonic Sequences: If a sequence $\{a_n\}$ is bounded and monotonic, then it converges.

1) List the terms of the following sequences:

   a) $\{a_n\} = \left\{\frac{n-1}{n+1}\right\}$

   b) $\{b_n\} = \left\{(-1)^{n+1}\left(\frac{n-1}{n}\right)\right\}$

   c) The recursively defined sequence $\{c_n\}$, where $c_1 = 2$ and $c_{n+1} = 2c_n + 1$.

2) If possible, find

$$\lim_{n\to\infty} \frac{3n}{\sqrt{n^2-2}}$$

3) Determine whether the following sequences converge or diverge.

a) $\{a_n\} = \{n + (-1)^n\}$

b) $\{b_n\} = \left\{\frac{3n^3}{3^n+3}\right\}$

4) Use the Squeeze Theorem to show that the sequence $\{a_n\} = \left\{\frac{\cos \pi n}{n^2}\right\}$ converges.

5) Find a sequence $\{a_n\}$ whose first five terms are

$$\frac{2}{3}, \frac{4}{9}, \frac{6}{27}, \frac{8}{81}, \frac{10}{243}, \dots$$

and determine whether the sequence you have chosen converges or diverges.

6) Determine an nth term for the sequence whose first six terms are

$$-\frac{2}{3}, \frac{3}{5}, -\frac{4}{9}, \frac{5}{17}, -\frac{6}{33}, \dots$$

and then decide whether the sequence converges or diverges.

7) Determine whether the sequence with the nth term $a_n = \frac{4n}{n+3}$ is monotonic. Discuss the boundedness of the sequence.

Homework for 9.1: #3, 5, 7, 9-12, 13, 16, 17, 20, 23, 33, 37, 38, 43, 49, 51, 52, 57