Section 8.8

Definition of Improper Integral with Infinite Integration Limits

1. If f is continuous on the interval $[a, \infty)$, then
$$\int_a^{\infty} f(x)\,dx = \lim_{b\to\infty} \int_a^b f(x)\,dx.$$
2. If f is continuous on the interval $(-\infty, b]$, then
$$\int_{-\infty}^{b} f(x)\,dx = \lim_{a\to-\infty} \int_a^b f(x)\,dx.$$
3. If f is continuous on the interval $(-\infty, \infty)$, then
$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{c} f(x)\,dx + \int_c^{\infty} f(x)\,dx$$
where c is any real number.

In the first two cases, the improper integral **converges** if the limit exists – otherwise, the improper integral **diverges**. In the third case, the improper integral on the left diverges if either of the improper integrals on the right diverges.

Definition of Improper Integral with Infinite Discontinuities

1. If f is continuous on the interval $[a, b)$ and has an infinite discontinuity at b, then
$$\int_a^b f(x)\,dx = \lim_{c\to b^-} \int_a^c f(x)\,dx.$$
2. If f is continuous on the interval $(a, b]$ and has an infinite discontinuity at a, then
$$\int_a^b f(x)\,dx = \lim_{c\to a^+} \int_c^b f(x)\,dx.$$
3. If f is continuous on the interval $[a, b]$, except for some c in (a, b) at which f has an infinite discontinuity then
$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx.$$

In the first two cases, the improper integral **converges** if the limit exists – otherwise, the improper integral **diverges**. In the third case, the improper integral on the left diverges if either of the improper integrals on the right diverges.

1) Evaluate $\int_1^{\infty} \frac{dx}{\sqrt{x}}$.

2) Evaluate the following improper integrals:

a) $\int_1^{\infty} \frac{dx}{x^{3/2}}$

b) $\int_{-\infty}^{0} e^{5x}dx$

3) Evaluate $\int_{1}^{\infty} xe^{-x}dx$

4) Evaluate $\int_{-\infty}^{\infty} \frac{dx}{x^2+4}$.

5) The energy, E, required to move a mass m from sea level at the equator at the surface of the earth (at a distance $r_E = 6378$ km) to a distance r from the center of the earth, is given by the equation

$$\int_{r_E}^{r} \frac{GMm}{r^2} dr$$

where $G = 6.673 \times 10^{-11}$ is the gravitational constant and $M = 5.9742 \times 10^{24}$ kg. Find the energy required for a 100 kg object to escape the gravitational pull of the earth (mathematically speaking, this is to move to infinite distance away from the earth).

6) Evaluate the following improper integrals.

a) $\int_{0}^{3} \frac{dx}{\sqrt{9-x^2}}$

b) $\int_{-3}^{1} \frac{dx}{(x-1)^2}$

c) $\int_{0}^{2} \frac{dx}{\sqrt[3]{x-1}}$

7) Evaluate $\int_{0}^{\infty} \frac{1}{\sqrt{x}(x+4)} dx$.

Homework for 8.8: #3, 9, 11, 21, 26, 27, 37, 41, 48