Section 8.7

L'Hôpital's Rule: Let f and g be functions that are differentiable on an open interval (a, b) containing c, except possibly at c itself. Assume that $g'(x) \neq 0$ for all x in (a, b), except possibly at c itself. If the limit of $f(x)/g(x)$ as x approaches c produces an indeterminate form $0/0$, then

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}$$

provided the limit on the right exists (or is infinite). This result also applies if the limit of $f(x)/g(x)$ as x approaches c produces any one of the indeterminate forms ∞/∞, $(-\infty)/\infty$, $\infty/(-\infty)$, or $(-\infty)/(-\infty)$.

1) Find the following limits:

   a) $\lim_{x \to 0} \frac{\sin x}{x}$

   b) $\lim_{t \to 0} \frac{5t^4 - 4t^2 - 1}{10 - t - 9t^3}$

2) Find $\lim_{x \to \infty} \frac{x}{e^x}$.

3) Find $\lim_{x\to\infty} \frac{x^2}{e^x}$.

4) Find $\lim_{x\to-\infty} xe^x$.

5) Find $\lim_{x\to\infty} x^{1/x}$.

6) Find $\lim_{x\to0^+} x^x$.

7) Find $\lim_{x\to0^+} \left(\frac{1}{x} - \frac{1}{\sin x}\right)$

Homework for 8.7: #9, 13, 15, 19, 29, 31, 43, 47, 55, 57, 58