Section 8.5

Decomposition of $N(x)/D(x)$ Into Partial Fractions

1. **Divide if improper:** If $N(x)/D(x)$ is an improper fraction (that is, if the degree of the numerator is greater than the or equal to the degree of the denominator), divide the denominator into the numerator to obtain
$$\frac{N(x)}{D(x)} = \text{(a polynomial)} + \frac{N_1(x)}{D(x)}$$
where $N_1(x)$ is less than the degree of $D(x)$. Then apply Steps 2, 3, and 4 to the proper rational expression $N_1(x)/D(x)$.

2. **Factor denominator:** Completely factor the denominator into factors of the form
$$(px+q)^m \quad \text{and} \quad (ax^2+bx+c)^n$$
where ax^2+bx+c is irreducible.

3. **Linear factors:** For each factor of the form $(px+q)^m$, the partial fraction decomposition must include the following sum of m fractions.
$$\frac{A_1}{px+q} + \frac{A_2}{(px+q)^2} + \cdots + \frac{A_m}{(px+q)^m}$$

4. **Quadratic factors:** For each factor of the form $(ax^2+bx+c)^n$, the partial fraction decomposition must include the following sum of n fractions.
$$\frac{B_1x+C_1}{ax^2+bx+c} + \frac{B_2x+C_2}{(ax^2+bx+c)^2} + \cdots + \frac{B_nx+C_n}{(ax^2+bx+c)^n}$$

1) $\int \frac{1}{4x^2-1}\,dx$

2) Find $\int \frac{5x-4}{x^3-4x^2+4x}\,dx$

3) Find $\int \frac{5x^2-3x+1}{x^3-2x^2+x-2}\,dx$

4) Find $\int \frac{x^3+2x^2+2x}{(x^2+1)^2}\,dx$

Homework for 8.5: #7, 9, 11, 15, 21, 22, 29, [redacted]