Section 7.6

Mass and Force: Mass and force are related by the equation $F=ma$.

Moments and Center of Mass: One Dimensional System: Let the point masses $m_1, m_2, \ldots, m_n$ be located at $x_1, x_2, \ldots, x_n$.

1. The **moment about the origin** is $M_0 = m_1 x_1 + m_2 x_2 + \cdots + m_n x_n.$
2. The **center of mass** is $\acute{x} = \frac{M_0}{m},$ where $m = m_1 + m_2 + \cdots + m_n$ is the **total mass** of the system.

Moment and Center of Mass: Two Dimensional System: Let the point masses $m_1, m_2, \ldots, m_n$ be located at $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n).$

1. The **moment of mass about the y -axis** is $M_y = m_1 x_1 + m_2 x_2 + \cdots + m_n x_n.$
2. The **moment of mass about the x -axis** is $M_x = m_1 y_1 + m_2 y_2 + \cdots + m_n y_n.$
3. The **center of mass** $(\acute{x}, \acute{y})$ (or **center of gravity**) is

   $$\acute{x} = \frac{M_y}{m} \text{ and } \acute{y} = \frac{M_x}{m}$$

   where $m = m_1 + m_2 + \cdots + m_n$ is the **total mass** of the system.

Moments and Center of Mass of a Planar Lamina: Let f and g be continuous functions such that $f(x) \geq g(x)$ on $[a, b]$, and consider the planar lamina of uniform density ρ bounded by the graphs of $y = f(x), y = g(x),$ and $a \leq x \leq b$.

1. The **moments about the x - and y -axes** are

   $$M_x = \rho \int_a^b \left[\frac{f(x) + g(x)}{2} \right] [f(x) - g(x)] dx \qquad M_y = \rho \int_a^b x[f(x) - g(x)] dx.$$

2. The **center of mass** $(\acute{x},\acute{y})$ is given by $\acute{x}=\frac{M_y}{m}$ and $\acute{y}=\frac{M_x}{m}$, where

$m=\rho\int_a^b[f(x)-g(x)]dx$ is the mass of the lamina.

Centroid of a Region in the Plane: The center of mass of a lamina with *uniform* density depends only on the shape of the lamina and not the density. The point $(\acute{x},\acute{y})$ is sometimes called the **centroid** of the region. It is found by assuming that $\rho=1$, and is given by

$$\acute{x}=\frac{1}{A}\int_a^b x[f(x)-g(x)]dx \wedge \acute{y}=\frac{1}{A}\int_a^b\left[\frac{f(x)+g(x)}{2}\right][f(x)-g(x)]dx$$

1) What is the mass (in kilograms) of an object that feels a force of 1000 newtons at sea level?

2) A point of mass 7 grams is located 4 cm to the left of the origin and a point of mass of 4 gm is located 8 cm to the right of the origin. The masses are connected by a thin, light rod. Find the center of mass of the system.

3) Find the center of mass of a system of point masses $m_1=3, m_2=5,$ and $m_3=1,$ located at $(4,-1),(1,3),$ and $(-2,2)$, respectively.

4) Find the center of mass of the lamina of uniform density ρ bounded by $f(x)=9-x^2$ and the x -axis.

5) Find the centroid of the region bounded by the graphs of $f(x)=9-x^2$ and $g(x)=6-2x$.

Homework for 7.6: #7, 9, 14, 19, 24