Section 7.4

Arc Length: Let $y = f(x)$ represent a smooth curve on the interval $[a, b]$. The arc length of f between a and b is

$$s = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$$

Similarly, for a smooth curve given by $x = g(y)$, the arc length of g between c and d is

$$s = \int_c^d \sqrt{1 + [g'(y)]^2}\, dy$$

Surface of Revolution: If the graph of a continuous function is revolved about a line, the resulting surface is a surface of revolution.

Area of a Surface of Revolution: Let $y = f(x)$ have a continuous derivative on the interval $[a, b]$. The area S of the surface of revolution formed by revolving the graph of f about a horizontal or vertical axis is

$$S = 2\pi \int_a^b r(x)\sqrt{1 + [f'(x)]^2} dx$$

where $r(x)$ is the distance between the graph of f and the axis of revolution. If $x = g(y)$ on the interval $[c, d]$, then the surface area is

$$S = 2\pi \int_c^d r(y)\sqrt{1 + [f'(y)]^2} dy$$

where $r(y)$ is the distance between the graph of g and the axis of revolution.

1) Find the arc length of the graph of $f(x) = \ln(\sec x)$ on the interval $\left[0, \frac{\pi}{4}\right]$.

2) Find the arc length of the graph of $x = \frac{2}{3}(y - 1)^{3/2}$ on the interval $[1, 4]$.

3) Find the area of the surface formed by revolving the graph of $f(x) = \sqrt{x}$ on the interval $[0, 1]$ about the x-axis.

4) Find the area of the surface formed by revolving the graph of $f(x) = 9 - x^2$ on the interval $[0, 3]$ about the y-axis.

Homework for 7.4: #4, 9, 15, 18, 25, 27, 34, 39, 44