

MATH 5370: FINAL PROJECTS ANNOUNCEMENTS

PROJECT ONE:

1. Device an efficient algorithm to invert an $n \times n$ lower triangular matrix A .
2. Code your algorithm and test it on
 - (a) matrix A with entries $a_{ij} = (i+j)^2$, $i \geq j$.
 - (b) matrix A with entries $a_{i,i-1} = 1$, $a_{i,i} = 1.5$.
3. Start with $n = 2, 3, \dots, 10$. Report how far you can reach with such computations?
4. Form the product $A \cdot A^{-1}$ to test the computed inverse.

PROJECT TWO:

1. Implement a code to realize the Gauss elimination algorithm to solve $Ax=b$ where A is a $n \times n$ matrix. Provide the factorization of $A=LU$ where L and U are lower and upper triangular matrices.

(a) You can start with A :

$$\begin{bmatrix} 1 & 6 & 0 \\ 2 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$$

and $b = [3 \ 1 \ 1]^T$.

(b) For the second matrix A :

$$\begin{bmatrix} -1 & 1 & 0 & -3 \\ 1 & 0 & 3 & 1 \\ 0 & 1 & -1 & -1 \\ 3 & 0 & 1 & 2 \end{bmatrix}$$

and the corresponding $b = [4 \ 0 \ 3 \ 1]^T$.

(**c**) A and b are described:

$$\begin{aligned} a_{i,j} &= 0.1 & \text{if } 1 \leq i \leq n-1, j=i. & & b_i &= 2.1-i & i, 1, 2, \dots, n. \\ a_{i,j} &= 1 & \text{if } 1 \leq i \leq n, j=n. & & b_n &= 2-n \\ a_{i,j} &= -1 & \text{if } i > j & & & & \\ a_{i,j} &= 0 & \text{otherwise.} & & & & \end{aligned}$$

Compare the Gauss elimination algorithm to: Gauss Elimination and directly solving the above system.

PROJECT THREE:

For a given $n \times n$ matrix,

1. Please compute the condition number $k(A)$ of the matrix $k(A) = \|A\| \|A^{-1}\|$ for the matrices described in projects One and Two.
2. Consider the matrix H with entries $H_{ij} = (i+j-1)^{-1}$, $i, j = 1, 2, \dots, n$.
 - I. Calculate the condition number of this matrix and the determinant of this matrix for values of $n=2, 3, \dots, 10$. How far can you get to compute this quantity?
 - II. Can you determine the dependence of the condition number on n ?

PROJECT FOUR:

Steepest descent v.s. conjugate gradient method for the two matrices described in Project Three. Take $b=[1,1,\dots,1]^T$.

PROJECT FIVE:

Generate probability distributions and calculate entities associated with them verify basic properties of distributions. **Project Proposal presentation is needed!** (Presentation date 28th October)

PROJECT SIX:

Financial Models using optimization. ([Here](#) is the Matlab analogue) **Project Proposal presentation is needed!** (Presentation date 28th October)

PROJECT SEVEN:

Genetic Algorithm. **Project Proposal presentation is needed!** (Presentation date 28th October)