Math 4329 **Midterm 02 Review List**

In addition to the in-class questions, homework and worksheet questions here are some additional problems:

1. Section 3.2: # 4
2. Section 3.4: # 8, # 9
3. Section 4.1: # 7, # 8, # 24, # 25, # 28
4. Section 4.3: # 10, # 14, #15, #16
5. Consider the following rootfinding methods to approximate $\alpha = 5^{1/3}$.

   (a) Write out the iteration formula for the **Secant Method** applied to the equation $x^3 - 5 = 0$ to find α. Simplify the formula as much as possible.
   Solution:
   $$x_{n+1} = \frac{x_n^2 x_{n-1} + x_n x_{n-1}^2 + 5}{x_n^2 + x_{n-1}x_n + x_{n-1}^2}$$
   Intermediate Steps:
   $$\begin{aligned} x_{n+1} &= x_n - (x_n^3 - 5)\frac{x_n - x_{n-1}}{x_n^3 - x_{n-1}^3} \\ &= x_n - \frac{(x_n^3 - 5)(x_n - x_{n-1})}{(x_n - x_{n-1})(x_n^2 + x_n x_{n-1} + x_{n-1}^2)} \\ &= x_n - \frac{x_n^3 - 5}{x_n^2 + x_n x_{n-1} + x_{n-1}^2} \\ &= \frac{x_n(x_n^2 + x_n x_{n-1} + x_{n-1}^2) - (x_n^3 - 5)}{x_n^2 + x_n x_{n-1} + x_{n-1}^2} \end{aligned}$$

   (b) Consider the fixed point iteration
   $$x_{n+1} = x_n + c(x_n^3 - 5)$$
   Find the values of c to ensure the convergence of the iterations generated by the above formula provided x_0 is chosen sufficiently close to α.
   Solution: $-1 < g'(\alpha) < 1$ **amounts to**
   $$\begin{aligned} -1 < 1 + 3c\alpha^2 < 1 \\ \frac{-2}{3\alpha^2} < c < 0. \end{aligned}$$
6. Consider the data $\{(1,1),(2,2),(3,5)\}$.

(a) Use Newton's divided difference formula to find the quadratic polynomial $p_2(x)$ that interpolates the above data. Find the expression in the simplest form.
Solution: You should get the polynomial $p_2(x) = x^2 - 2x + 2$

(b) Use Lagrange's formula to find $p_2(x)$ and show that you got the same result as in (a).
Useful Tip: We know that the polynomial of degree 2 passing through 3 points $\{(1,1),(2,2),(3,5)\}$ **will always be unique so if the polynomial you obtained passes through the three given points that means it is the right one.**

7. Determine the values of $a,\ b$, and c so that the following is a cubic spline function on [0, 3].

$$s(x) = \begin{cases} x^3 & \text{if } 0 \le x \le 2, \\ -0.5(x-1)^3 + a(x-1)^2 + b(x-1) + c & \text{if } 2 \le x \le 3. \end{cases}$$

Solution: a=7.5, b=-1.5, c=2.5.

Useful Tip: Check you answer by plugging in the values of $a,\ b,\ c$ **into the three equations obtained.**