Math 4329: Errors in Solving Linear Systems **Due on: November 10, 2016**

Name: _____

Student ID #: _____

Class Time: _____

1. Consider the following linear system:

$$\begin{aligned} x + y &= 0 \\ x + \frac{401}{400}y &= 20 \end{aligned}$$

(a) Obtain the solution by a direct inversion of the coefficient matrix and assuming that we are using a computer with five significant digits. Verify that the solution obtained is the exact solution $\mathbf{x}^* = [-8000, 8000]^T$.

(b) Compare the above exact solution to one obtained on a computer with four digits of significance.

(c) Compare the above exact solution to one obtained on a computer with three digits of significance. Please remark on the existence of the solution.
Hint: The columns are multiples of each other.

2. Consider the following linear system:

$$\begin{aligned} x - \frac{800}{801}y &= 10 \\ -x + y &= 50 \end{aligned}$$

(a) Verify that the exact solution is $\mathbf{x}^* = [48010, 48060]^T$.

(b) Obtain the solution by a direct inversion of the coefficient matrix and assuming that we are using a computer with eight significant digits, compare it with the exact solution $\mathbf{x}^*$.

(c) Compare the solution to one obtained on a computer with four digits of significance and with three digits of significance. Remark on how large the error is for both these cases.

3. Solve the following linear system:

$$x + y = 2$$
$$x + 1.0001y = 2 + \alpha.$$

Here the number α assumes the following three values: 0, 10^{-3} and 10^{-4}. Can you explain the significant difference in the solutions ?

Solution:
$\alpha = 0; \quad x = 2; \quad y = 0.$
$\alpha = 10^{-3}; \quad x = 0.999999999996362; \quad y = 1.000000000003638,$
$\alpha = 10^{-4}; \quad x = 1.899999999997817; \quad y = 0.100000000002183.$