Math 4329 Gaussian Elimination and LU Factorization **Due on: November 15, 2016**

Name: _____

Student ID #: _____

Class Time: _____

1. Provide the LU decomposition of the following matrices:

(a)

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 1 & 3 & 9 \\ 1 & 4 & 16 \end{bmatrix}$$

(b)

$$A = \begin{bmatrix} 1 & -1 & 2 \\ -1 & 5 & 4 \\ 2 & 4 & 29 \end{bmatrix}$$

2. Gaussian Elimination

(a) Solve the linear system $Ax = b$ by using Gaussian Elimination without pivoting where:

$$A = \begin{bmatrix} 2 & 1 & -1 \\ 4 & 0 & -1 \\ -8 & 2 & 2 \end{bmatrix}, b = \begin{bmatrix} 6 \\ 6 \\ -8 \end{bmatrix}$$

(b) Use Gaussian elimination with back substitution to solve the system:

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 4 \\ 3x_1 + 4x_2 + x_3 &= 6 \\ 2x_1 + 5x_2 + 7x_3 &= 2 \end{aligned}$$

Please specify the multipliers m_{21}, m_{31} and m_{32}.

(c) Use the multipliers from the previous part (b) to form the LU factorization of the coefficient matrix of the linear system.

3. (a) Use Gaussian Elimination to solve the following system after each calculation, round the result to three significant digits:

$$0.143x_1 + 0.357x_2 + 2.01x_3 = -5.173$$
$$-1.31x_1 + 0.911x_2 + 1.99x_3 = -5.458$$
$$11.2x_1 - 4.3x_2 - 0.605x_3 = 4.415$$

(b) Compare the solution to the following one:

$$x_1 = 1, \quad x_2 = 2 \quad \text{and} \quad x_3 = -3.$$

Why does the difference arise ?

(c) Explain what can be done to overcome this issue.