Math 4329 MATRIX ARITHMETIC

Name: ____

Student ID #: ______________________________________

Class Time: ____

1. Compute the inverse of the Hilbert Matrix of order 2, 3, 4.

2. Transpose of a matrix A is A^T obtained by interchanging the rows and the columns of A.

(a) Find out all the numeric entries of A satisfying the following:

$$A = \begin{bmatrix} a & b & 1 \\ & & \\ 2 & c & d \end{bmatrix} = \begin{bmatrix} 5 & 6 & 1 \\ & & \\ 2 & 2 & a+1 \end{bmatrix}$$

(b) What is the transpose of A i.e., A^T?

(c) What is the inverse of A ?

(d) Find out A^*A^T and $A^{T*}A$.

3. Let

$$A = \begin{bmatrix} 2 & 3 & 2 \\ -1 & 2 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 2 & -1 \\ 3 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 6 & 3 \\ 2 & -1 & 2 & 4 \\ 3 & 1 & 12 & 7 \end{bmatrix}.$$

- Compute A*B, A*C.

- Is it possible to compute B*A, C*A as well ? If yes, please compute it. If not, please provide the reason.

4. Let

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 3/4 & -1/2 & 1/4 \\ -1/2 & 1 & -1/2 \\ 1/4 & -1/2 & 3/4 \end{bmatrix}$$

(a) Using Gaussian Elimination, find the inverse of A.

(b) Verify that it is B by computing A*B and B*A.

5. Simplify the following matrix expressions to obtain a single matrix:

(a)

$$2\begin{bmatrix} 1 & 0 \\ -1 & 3 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} * \begin{bmatrix} 2 & 3 \\ 1 & 3 \end{bmatrix}$$

(b) $A = I_3 - 2x * x^T, \quad x^T = [1/3, 2/3, 2/3], \qquad B = A^2.$

6. Solve the system below:

$$\begin{aligned} 4x_1 + 3x_2 + 2x_3 + x_4 &= 1 \\ 3x_1 + 4x_2 + 3x_3 + 2x_4 &= 1 \\ 2x_1 + 3x_2 + 4x_3 + 3x_4 &= -1 \\ x_1 + 2x_2 + 3x_3 + 4x_4 &= -1 \end{aligned}$$

by converting it to the form $Ax = b$ and using Gaussian Elimination. Give a count of the number of mathematical operations involved at each step.

7. List the following statements as true or false. If false give an example to support your answer.

(a) If A is invertible then det(A)$\neq 0$.

(b) det(AB)=det(A)det(B).

(c) A is a 3×4 matrix does its inverse exist ?

(d) det(A)=det(A^T)