**Math 4329 Midterm Review** **February 7, 2017**

**1.** This question is on the Taylor polynomial.

(a) Find the Taylor Polynomial $\mathrm{p}_1(\mathrm{x})$ for $\mathrm{f(x)}= \log(1+e^{-x})$ about the point $a=0$.

(b) Bound the error $|f(x)-p_1(x)|$ using the Taylor Remainder $R_1(x)$ on $[-1,1]$ for the choice of $f(x)$ as described above.

**2.** Let $p_n(x)$ be the Taylor Polynomial of degree $n$ of $f(x)=\log(x+3)$ about $a=0$. How large should $n$ be so that $|f(x)-p_n(x)|<10^{-3}$ for $-1 \le x \le 0.5$ ?

**3.** (10 pts) This question is related to floating-point numbers.

(a) Determine the number $x$ that has the following binary format:

$$(1111\ 1001\ 101)_2$$

(b) Furthermore, recall the **double** precision representation for any number y is

$$y=\sigma \cdot (1.a_1a_2a_3\cdots a_{52})\cdot 2^{E-1023},\ \text{where } E=(c_1c_2c_3\cdots c_{11})_2.$$

Please express the number $x$ obtained above in its double precision representation.

**4.** This question is concerned with the loss-of-significance errors.

(a) Explain the difficulty in the evaluation of $f(x)=x\left(\sqrt{x}-\sqrt{x-1}\right)$ for increasing values of $x=10^2+1,\ 10^4+1,\ 10^6+1,\ 10^8+1,\ 10^{10}+1$.

(b) Reformulate $f(x)$ to avoid the above difficulty in (a). Simplify the expression as much as possible.

**5.** Use the Taylor Polynomial of degrees $1,\ 2$ and 3 for $f(x)=x+1^{1/3}$ to compute the value of $2^{1/3}$ about 0.

**6.** Calculate the error, the relative error and the number of significant digits in the approximation $x_A=22/7$ to $x_T=3.141593$.