Math 4329 EXAM 02 REVIEW

1. Check whether the following function is a spline:

$$f(x) = \begin{cases} x^3 & 0 \le x \le 1, \\ 2x - 1 & 1 < x < 2, \\ 3x^2 - 2 & 2 \le x \le 3. \end{cases}$$

Also see: problems 12-16, page 158.

2. Without computing the interpolating polynomial $p(x)$, estimate the error in interpolating the $f(x)$ where $f(x) = \sin(\pi x)$, at $0, 0.5, 1, -1$.

3. (**Quadrature Rules**)

(a) Find c_1 and c_2 in the following quadrature formula:

$$\int_{-1}^{1} f(x)dx \approx c_1 f(-1) \ + \ c_2 f(1)$$

so that is exact for all polynomials of the largest degree possible. What is the degree of precision for this formula?

(b) What is the error in using the 4-point trapezoidal/Simpson's rule applied to

$$f(x) = e^x \cos(4x)$$

on the interval $(-\pi, \pi)$?

(c) Consider the task of evaluating

$$I = \int_0^2 e^{-x^2}\, dx$$

using the trapezoidal rule $T_n(f)$.
How large should n be chosen in order to ensure that

$$|E_n^T(f)| \le 10^{-4}?$$

(d) i. Approximate $\int_{-1}^{1} x^8\, dx$ using the two point Gaussian quadrature rule with nodes $\pm 3^{-1/2}$ and weights 1.

ii. Calculate the exact integral $\int_{-1}^{1} x^8\, dx$ and compare the error between the true value and the approximation obtained in 1(b)i.

Also see: problems 8, 9 page 230.

4. Consider the following table:

x	0.3	0.4	0.5	0.6	0.7
$f(x)$	1.5	2	5	7	12

Let $x_{i+1} = x_i + h, \quad i = 0, \cdots, 3.$

(a) Approximate $f'(0.5)$ using $D_h^+ f(0.5)$ and $h = 0.1$.

(b) Compute $D_h^{(2)} f(0.5)$ using the Central Difference Formula and step size $h = 0.2$.
Note: You may use the following formula for Central Difference Formula:

$$D_h^{(2)} f(x_1) = \frac{D_h^+ f(x_1) - D_h^- f(x_1)}{h}$$

(c) Compare the answer from (b) with the following approximation:

$$D_h^{(2)} f(x_1) = \frac{f(x_2) - 2f(x_1) + f(x_0)}{h^2},$$

with $x_1 = 0.5, \; h = 0.2$.