You may use a calculator and the formula sheets at the end of the exam. **You must show all of your work to receive full credit for a problem. Correct answers without supporting work will receive no credit!**

1) Find the indefinite integral.
   $$\int x^3\sqrt{4-x^2}\,dx$$

2) Find the limit.
   $$\lim_{x\to\infty} x^{1/x}$$

3) A cylindrical tank is 10 feet high with a diameter of 4 feet. The tank is buried upright so that the top of the tank is 2 feet underground. The tank is full of oil with a weight density of 100 pounds per cubic foot. How much work is required to pump all of the oil from the tank to the ground level?

4) Evaluate the improper integral, if possible.
   $$\int_0^\infty \frac{1}{(3x+1)^3}\,dx$$

5) Find the indefinite integral.
   $$\int (2x+1)e^{3x}\,dx$$

6) Find the interval of convergence for the power series. $\sum_{n=0}^{\infty}\frac{(-1)^n(x-3)^n}{3n+2}$

   Don't forget to check the endpoints of the interval!

7) Consider the region bounded by $y=1-\sqrt{x^3}$, the x -axis, and the y -axis. Find the volume of the solid obtained by rotating this region about the x -axis.

8) Write the series using sigma notation, then find the sum of the series, if possible.
   $$\frac{-2}{3}+\frac{4}{9}-\frac{8}{27}+\frac{16}{81}-\ldots$$

Extra Credit: Find the following integral (all work must be shown, including any factoring that may be necessary):
$$\int \frac{6x^2-31x+57}{x^3-7x^2+19x-13}\,dx$$

DERIVATIVES AND INTEGRALS

Basic Differentiation Rules

1. $\frac{d}{dx}[cu] = cu'$
2. $\frac{d}{dx}[u \pm v] = u' \pm v'$
3. $\frac{d}{dx}[uv] = uv' + vu'$
4. $\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{vu' - uv'}{v^2}$
5. $\frac{d}{dx}[c] = 0$
6. $\frac{d}{dx}[u^n] = nu^{n-1}u'$
7. $\frac{d}{dx}[x] = 1$
8. $\frac{d}{dx}[|u|] = \frac{u}{|u|}(u'), \quad u \neq 0$
9. $\frac{d}{dx}[\ln u] = \frac{u'}{u}$
10. $\frac{d}{dx}[e^u] = e^u u'$
11. $\frac{d}{dx}[\log_a u] = \frac{u'}{(\ln a)u}$
12. $\frac{d}{dx}[a^u] = (\ln a)a^u u'$
13. $\frac{d}{dx}[\sin u] = (\cos u)u'$
14. $\frac{d}{dx}[\cos u] = -(\sin u)u'$
15. $\frac{d}{dx}[\tan u] = (\sec^2 u)u'$
16. $\frac{d}{dx}[\cot u] = -(\csc^2 u)u'$
17. $\frac{d}{dx}[\sec u] = (\sec u \tan u)u'$
18. $\frac{d}{dx}[\csc u] = -(\csc u \cot u)u'$
19. $\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1 - u^2}}$
20. $\frac{d}{dx}[\arccos u] = \frac{-u'}{\sqrt{1 - u^2}}$
21. $\frac{d}{dx}[\arctan u] = \frac{u'}{1 + u^2}$
22. $\frac{d}{dx}[\text{arccot}\, u] = \frac{-u'}{1 + u^2}$
23. $\frac{d}{dx}[\text{arcsec}\, u] = \frac{u'}{|u|\sqrt{u^2 - 1}}$
24. $\frac{d}{dx}[\text{arccsc}\, u] = \frac{-u'}{|u|\sqrt{u^2 - 1}}$
25. $\frac{d}{dx}[\sinh u] = (\cosh u)u'$
26. $\frac{d}{dx}[\cosh u] = (\sinh u)u'$
27. $\frac{d}{dx}[\tanh u] = (\text{sech}^2\, u)u'$
28. $\frac{d}{dx}[\coth u] = -(\text{csch}^2\, u)u'$
29. $\frac{d}{dx}[\text{sech}\, u] = -(\text{sech}\, u \tanh u)u'$
30. $\frac{d}{dx}[\text{csch}\, u] = -(\text{csch}\, u \coth u)u'$
31. $\frac{d}{dx}[\sinh^{-1} u] = \frac{u'}{\sqrt{u^2 + 1}}$
32. $\frac{d}{dx}[\cosh^{-1} u] = \frac{u'}{\sqrt{u^2 - 1}}$
33. $\frac{d}{dx}[\tanh^{-1} u] = \frac{u'}{1 - u^2}$
34. $\frac{d}{dx}[\coth^{-1} u] = \frac{u'}{1 - u^2}$
35. $\frac{d}{dx}[\text{sech}^{-1} u] = \frac{-u'}{u\sqrt{1 - u^2}}$
36. $\frac{d}{dx}[\text{csch}^{-1} u] = \frac{-u'}{|u|\sqrt{1 + u^2}}$

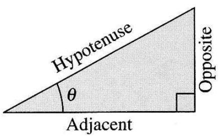
Basic Integration Formulas

1. $\int kf(u)\, du = k\int f(u)\, du$
2. $\int [f(u) \pm g(u)]\, du = \int f(u)\, du \pm \int g(u)\, du$
3. $\int du = u + C$
4. $\int a^u\, du = \left(\frac{1}{\ln a}\right)a^u + C$
5. $\int e^u\, du = e^u + C$
6. $\int \sin u\, du = -\cos u + C$
7. $\int \cos u\, du = \sin u + C$
8. $\int \tan u\, du = -\ln|\cos u| + C$
9. $\int \cot u\, du = \ln|\sin u| + C$
10. $\int \sec u\, du = \ln|\sec u + \tan u| + C$
11. $\int \csc u\, du = -\ln|\csc u + \cot u| + C$
12. $\int \sec^2 u\, du = \tan u + C$
13. $\int \csc^2 u\, du = -\cot u + C$
14. $\int \sec u \tan u\, du = \sec u + C$
15. $\int \csc u \cot u\, du = -\csc u + C$
16. $\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin \frac{u}{a} + C$
17. $\int \frac{du}{a^2 + u^2} = \frac{1}{a} \arctan \frac{u}{a} + C$
18. $\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \text{arcsec} \frac{|u|}{a} + C$

TRIGONOMETRY

Definition of the Six Trigonometric Functions

Right triangle definitions, where $0 < \theta < \pi/2$.

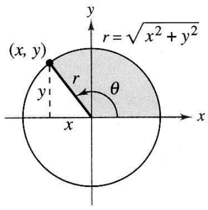


$\sin\theta = \dfrac{\text{opp}}{\text{hyp}}$ $\quad \csc\theta = \dfrac{\text{hyp}}{\text{opp}}$

$\cos\theta = \dfrac{\text{adj}}{\text{hyp}}$ $\quad \sec\theta = \dfrac{\text{hyp}}{\text{adj}}$

$\tan\theta = \dfrac{\text{opp}}{\text{adj}}$ $\quad \cot\theta = \dfrac{\text{adj}}{\text{opp}}$

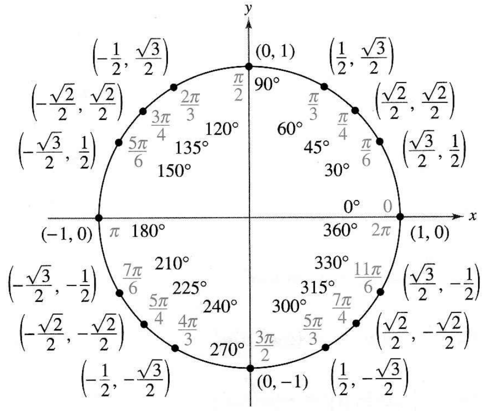
Circular function definitions, where θ is any angle.



$\sin\theta = \dfrac{y}{r}$ $\quad \csc\theta = \dfrac{r}{y}$

$\cos\theta = \dfrac{x}{r}$ $\quad \sec\theta = \dfrac{r}{x}$

$\tan\theta = \dfrac{y}{x}$ $\quad \cot\theta = \dfrac{x}{y}$



Reciprocal Identities

$\sin x = \dfrac{1}{\csc x}$ $\quad \sec x = \dfrac{1}{\cos x}$ $\quad \tan x = \dfrac{1}{\cot x}$

$\csc x = \dfrac{1}{\sin x}$ $\quad \cos x = \dfrac{1}{\sec x}$ $\quad \cot x = \dfrac{1}{\tan x}$

Tangent and Cotangent Identities

$\tan x = \dfrac{\sin x}{\cos x}$ $\quad \cot x = \dfrac{\cos x}{\sin x}$

Pythagorean Identities

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$ $\quad 1 + \cot^2 x = \csc^2 x$

Cofunction Identities

$\sin\left(\dfrac{\pi}{2} - x\right) = \cos x$ $\quad \cos\left(\dfrac{\pi}{2} - x\right) = \sin x$

$\csc\left(\dfrac{\pi}{2} - x\right) = \sec x$ $\quad \tan\left(\dfrac{\pi}{2} - x\right) = \cot x$

$\sec\left(\dfrac{\pi}{2} - x\right) = \csc x$ $\quad \cot\left(\dfrac{\pi}{2} - x\right) = \tan x$

Reduction Formulas

$\sin(-x) = -\sin x$ $\quad \cos(-x) = \cos x$

$\csc(-x) = -\csc x$ $\quad \tan(-x) = -\tan x$

$\sec(-x) = \sec x$ $\quad \cot(-x) = -\cot x$

Sum and Difference Formulas

$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$

$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$

$\tan(u \pm v) = \dfrac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$

Double-Angle Formulas

$\sin 2u = 2 \sin u \cos u$

$\cos 2u = \cos^2 u - \sin^2 u = 2\cos^2 u - 1 = 1 - 2\sin^2 u$

$\tan 2u = \dfrac{2\tan u}{1 - \tan^2 u}$

Power-Reducing Formulas

$\sin^2 u = \dfrac{1 - \cos 2u}{2}$

$\cos^2 u = \dfrac{1 + \cos 2u}{2}$

$\tan^2 u = \dfrac{1 - \cos 2u}{1 + \cos 2u}$

Sum-to-Product Formulas

$\sin u + \sin v = 2\sin\left(\dfrac{u+v}{2}\right)\cos\left(\dfrac{u-v}{2}\right)$

$\sin u - \sin v = 2\cos\left(\dfrac{u+v}{2}\right)\sin\left(\dfrac{u-v}{2}\right)$

$\cos u + \cos v = 2\cos\left(\dfrac{u+v}{2}\right)\cos\left(\dfrac{u-v}{2}\right)$

$\cos u - \cos v = -2\sin\left(\dfrac{u+v}{2}\right)\sin\left(\dfrac{u-v}{2}\right)$

Product-to-Sum Formulas

$\sin u \sin v = \dfrac{1}{2}[\cos(u - v) - \cos(u + v)]$

$\cos u \cos v = \dfrac{1}{2}[\cos(u - v) + \cos(u + v)]$

$\sin u \cos v = \dfrac{1}{2}[\sin(u + v) + \sin(u - v)]$

$\cos u \sin v = \dfrac{1}{2}[\sin(u + v) - \sin(u - v)]$

SUMMARY OF TESTS FOR SERIES

Test	Series	Condition(s) of Convergence	Condition(s) of Divergence	Comment
nth-Term	$\sum_{n=1}^{\infty} a_n$		$\lim_{n\to\infty} a_n \neq 0$	This test cannot be used to show convergence.
Geometric Series	$\sum_{n=0}^{\infty} ar^n$	$\lvert r\rvert < 1$	$\lvert r\rvert \geq 1$	Sum: $S = \frac{a}{1-r}$
Telescoping Series	$\sum_{n=1}^{\infty} (b_n - b_{n+1})$	$\lim_{n\to\infty} b_n = L$		Sum: $S = b_1 - L$
p-Series	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	$p > 1$	$0 < p \leq 1$	
Alternating Series	$\sum_{n=1}^{\infty} (-1)^{n-1} a_n$	$0 < a_{n+1} \leq a_n$ and $\lim_{n\to\infty} a_n = 0$		Remainder: $\lvert R_N\rvert \leq a_{N+1}$
Integral (f is continuous, positive, and decreasing)	$\sum_{n=1}^{\infty} a_n$, $a_n = f(n) \geq 0$	$\int_1^{\infty} f(x)\,dx$ converges	$\int_1^{\infty} f(x)\,dx$ diverges	Remainder: $0 < R_N < \int_N^{\infty} f(x)\,dx$
Root	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \sqrt[n]{\lvert a_n\rvert} < 1$	$\lim_{n\to\infty} \sqrt[n]{\lvert a_n\rvert} > 1$ or $= \infty$	Test is inconclusive if $\lim_{n\to\infty} \sqrt[n]{\lvert a_n\rvert} = 1$.
Ratio	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \left\lvert\frac{a_{n+1}}{a_n}\right\rvert < 1$	$\lim_{n\to\infty} \left\lvert\frac{a_{n+1}}{a_n}\right\rvert > 1$ or $= \infty$	Test is inconclusive if $\lim_{n\to\infty} \left\lvert\frac{a_{n+1}}{a_n}\right\rvert = 1$.
Direct Comparison ($a_n, b_n > 0$)	$\sum_{n=1}^{\infty} a_n$	$0 < a_n \leq b_n$ and $\sum_{n=1}^{\infty} b_n$ converges	$0 < b_n \leq a_n$ and $\sum_{n=1}^{\infty} b_n$ diverges	
Limit Comparison ($a_n, b_n > 0$)	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \frac{a_n}{b_n} = L > 0$ and $\sum_{n=1}^{\infty} b_n$ converges	$\lim_{n\to\infty} \frac{a_n}{b_n} = L > 0$ and $\sum_{n=1}^{\infty} b_n$ diverges	