Math 1312 **Practice Exam 1**

There will be 5-6 problems on the actual exam. All of them will be similar to the problems shown here. A copy of the formula sheet from the front of the book will be attached to the exam. You may also use your calculator, but you must show all of your work to receive full or partial credit for a problem.

1) Find the following integrals. You may use any of the formulas on the sheet, but if you do use a formula be sure to state which one you are using and clearly identify u and a.

   a) $\int \frac{3}{x\sqrt{x^2-9}}\,dx \quad \left(\operatorname{arcsec}\frac{|x|}{3} + C\right)$

   b) $\int \frac{2}{x^2-2x+5}\,dx \quad \left(\arctan\left(\frac{x-1}{2}\right) + C\right)$

   c) $\int \frac{6}{2e^x+5}\,dx \quad \left(-\frac{6}{5}\ln(2+5e^{-x}) + C, \text{ or } -\frac{6}{5}\ln(2e^x+5) + \frac{6}{5}x + C\right)$

   d) $\int \frac{1}{\sin\theta+1}\,d\theta \quad (\tan\theta - \sec\theta + C)$

2) Find each integral using integration by parts, or another method if it's easier (state which method you are using).

   a) $\int \ln x\,dx \quad (x\ln x - x + C)$

   b) $\int x\cos x\,dx \quad (x\sin x + \cos x + C)$

   c) $\int \frac{x}{\sqrt{4x+5}}\,dx$

   $$\left(\text{Parts: } \frac{1}{2}x\sqrt{4x+5} - \frac{1}{12}(4x+5)^{3/2}, \qquad \text{Substitution: } \frac{1}{24}(4x+5)^{3/2} - \frac{5}{8}\sqrt{4x+5}\right)$$

   d) $\int \theta\csc\theta\cot\theta\,d\theta \quad (-\theta\csc\theta - \ln|\csc\theta + \cot\theta| + C)$

3) Find each trigonometric integral.

   a) $\int \sin^3 x\cos^4 x\,dx \quad \left(-\frac{1}{5}\cos^5 x + \frac{1}{7}\cos^7 x + C\right)$

   b) $\int \tan\theta\sec^4\theta\,d\theta \quad \left(\frac{1}{4}\sec^4\theta + C\right)$

4) Find the following integrals using an appropriate trigonometric substitution.

a) $\int \frac{\sqrt{x^2-9}}{x} dx$ $\left(\sqrt{x^2-9} - 3\,\text{arcsec}\frac{x}{3} + C\right)$

b) $\int \frac{\sqrt{9-x^2}}{x} dx$ $\left(-3\ln\left|\frac{3+\sqrt{9-x^2}}{x}\right| + \sqrt{9-x^2} + C\right)$

c) $\int \frac{1}{(x^2+16)^{3/2}} dx$ $\left(\frac{x}{16\sqrt{x^2+16}} + C\right)$

5) Find the following using the method of partial fractions (or any other method if it's easier).

a) $\int \frac{3x+4}{x^2-2x+1} dx$ $\left(3\ln|x-1| - \frac{7}{x-1} + C\right)$

b) $\int \frac{x^2+2x}{x^3-x^2+x-1} dx$ $\left(\frac{3}{2}\ln|x-1| - \frac{1}{4}\ln(x^2+1) + \frac{3}{2}\arctan x + C\right)$

6) Find the limit.

$$\lim_{x \to 0^+} x^x$$

(The limit is 1)

7) Evaluate the improper integral, or state that it diverges.

$$\int_1^3 \frac{1}{\sqrt[3]{x-1}} dx$$

(The "problem" value is $x = 1$ since the function is undefined there. So, we rewrite the integral as $\lim_{b\to 1^+} \int_b^3 \frac{1}{\sqrt[3]{x-1}} dx = \lim_{b\to 1^+} \left[\frac{3}{2}(x-1)^{2/3}\right]_b^3 = \lim_{b\to 1^+} \left[\frac{3}{2}(2)^{2/3} - \frac{3}{2}(b-1)^{2/3}\right] = \frac{3}{2}(2)^{2/3}$)