The Role of Counterintuitive Examples in Statistics Education

by

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The Role of Counterintuitive Examples

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The purpose of this study was to develop a theoretical model for the use of counterintuitive examples in the introductory non–calculus-based statistics course at the college level. While intuition and misconceptions continue to be of great interest to mathematics and science educators, there has been little research, much less consensus or even internal consistency, in statistics curriculum development concerning the role of examples with counterintuitive results. Because the study intended to provide educators with useful connections to content, instructional methods (e.g., cooperative learning) and learning theory constructs that have been successfully used in mathematics or science education, the model that emerged was organized around a typical syllabus of topics.

The study critiqued and then reconciled "Traditional" and "Alternative" perspectives. The Traditional Position attempts to minimize possible confusion and frustration by avoiding such examples, while the Alternative Position uses them to motivate and engage students in critical thinking, active learning, metacognition, communication of their ideas, real-world problem solving and exploration, reflection on the nature and process of statistics, and other types of activities encouraged by current reform movements.

The study delineated specific criteria and conditions for selecting and using counterintuitive examples to achieve numerous cognitive and affective objectives. Examples explored include the Monty Hall problem, Simpson's Paradox, the birthday problem, de Méré's Paradox, the Classification Paradox, the Inspection Paradox, and required sample size. The study connected many of

these examples (especially Simpson's Paradox) with other counterintuitive examples, with known probability or statistics misconceptions many students have, with representations from other branches of mathematics, and with the constructivist paradigm.

Problematic issues addressed include difficulty in constructing assessment instruments and a multiplicity of terminologies and typologies. Additional directions for research were suggested, including several empirical investigations of various facets of the model. The connections, examples, and representations presented should be extremely useful for teachers of statistics, but should also enrich the pedagogy of teachers of other courses.

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