Math 5370   **ALGEBRAIC STRUCTURES**   Spring 2020
Dr. Duval   **Homework 7**
due Thursday, March 26

1. Define, for any real numbers a and b,

$$S_{a,b} = \begin{pmatrix} a & -b \\ b & a \end{pmatrix}.$$

Also define

$$F = \{S_{a,b}\colon a \in \mathbf{Q},\ b \in \mathbf{Q}\} \subseteq M_2(\mathbf{Q}),$$

a subset of $M_2(\mathbf{Q})$ (the set of 2×2 rational matrices).

Prove that F is a **subring** of $M_2(\mathbf{Q})$, with the usual matrix addition and multiplication.

2. Prove that F from problem **1.** is a field.

3. Define, for any real numbers a and b,

$$T_{a,b} = \begin{pmatrix} a + b & b \\ b & a - b \end{pmatrix}.$$

Also define

$$D = \{T_{a,b}\colon a \in \mathbf{Z},\ b \in \mathbf{Z}\} \subseteq M_2(\mathbf{Z}),$$

a subset of $M_2(\mathbf{Z})$ (the set of 2×2 integer matrices).

Prove that D is a **subring** of $M_2(\mathbf{Z})$, with the usual matrix addition and multiplication.

4. Prove that D from problem **3.** is an integral domain. (**Hint:** Remember that $\sqrt{2}$ is irrational.)