Math 5370 | **ALGEBRAIC STRUCTURES** | Spring 2020

Dr. Duval

Homework 3

due Thursday, February 13

1. Define a mapping $\phi\colon \mathbf{Z}_n \to \mathbf{Z}_n$ by $\phi([a]) = [n-a]$, for $0 \le a < n$. Prove that ϕ is an isomorphism from the additive group $\mathbf{Z}_n$ to itself.

2. Consider the additive groups $\mathbf{Z}_{14}$ and $\mathbf{Z}_7$, and define $\phi\colon \mathbf{Z}_{14} \to \mathbf{Z}_7$ by

$$\phi([x]_{14}) = [3x]_7.$$

Prove that ϕ is a homomorphism and find $\ker \phi$. Is ϕ an epimorphism? Is ϕ a monomorphism?

3. Recall that in Homework 2 you proved that

$$H = \left\{ \begin{bmatrix} 1 & b \\ 0 & 1 \end{bmatrix} : b \in \mathbf{R} \right\}.$$

is a subgroup of $GL(2, \mathbf{R})$. Prove that H is isomorphic to $(\mathbf{R}, +)$, the additive group of the real numbers. (Note that the group operation for H is **multiplication**, but the group operation for $(\mathbf{R}, +)$ is **addition**.)

4. Recall that the magnitude $|a+bi|$ of a complex number $a+bi$ is given by $|a+bi| = \sqrt{a^2+b^2}$.

(a) Prove that $\phi(a+bi) = |a+bi|$ is a homomorphism from the multiplicative group of the non-zero complex numbers $\mathbf{C} - \{0\}$ to the multiplicative group of positive real numbers $\mathbf{R}^+$.

(b) Draw the location in the complex plane of all the elements of $\ker \phi$.