Math 5370 **GAME THEORY** Spring 2019
Dr. Duval **Homework**
Wednesday, March 27

Follow the separate general guidelines for Parts A,B,C. Be sure to include and label *all four* standard parts (a), (b), (c), (d) of Part A in what you hand in.

General-sum games with more than two players
Section 4.3

A: Reading questions. Due by 2pm, Sun., 31 Mar.

1. Demonstrate your understanding of the formal definition of **utility function** at the beginning of the section by describing a possible set of utility functions for the game One Hundred Gnus and a Lioness.
2. Show that Definitions 4.3.4 and 4.3.5 and Lemma 4.3.7 are generalizations of the corresponding definitions and result in a two-player game.
3. Verify some details of the Pollution game:
   (a) The two pure equilibria described just after Remark 4.3.8 are indeed each equilibria.
   (b) "All three firms would prefer an of the asymmetric equilibria, but cannot unilaterally transition to these equilibria."
   (c) If player III purifies, then it is a best response for each of player I and II to purify with probability 2/3 and pollute with probability 1/3.
   (d) There is no equilibrium with two pure and one non-pure strategy.

B: Warmup exercises. For you to present in class. Due by the end of class Mon., 1 Apr.

Exercise 4.8

Potential games
Section 4.4

A: Reading questions. Due by 2pm, Tue., 2 Apr.

1. Let's try to make an example of the Congestion Game on the graph in Figure 4.10. For the road from a to b, let the cost function $c_{a,b}(n)$ to each driver on this road if there are n drivers be given by $c_{a,b}(1) = 1, c_{a,b}(2) = 3, c_{a,b}(3) = 5$ (so if there are two drivers on this road, each will pay a cost of 3); similarly, for the road from a to c, set $c_{b,c}(1) = 1, c_{b,c}(2) = 4, c_{b,c}(3) = 9$, and for the road from a to c, set $c_{a,c}(1) = 8, c_{a,c}(2) = 9, c_{a,c} = 10$. (Note that this is a more complicated cost function than given in the caption for Figure 4.10.)
   (a) Find the cost of each driver for each of the two (sub)figures in Figure 4.10.
   (b) Will the green driver want to make the switch from the left (sub)figure to the right (sub)figure?
   (c) Will the red driver want to change their path from b, c, a to the more direct path b, a?
   (d) Will the purple driver want to change their path from a, b, c to the more direct path a, c?
   (e) Use equation (4.5) to compute ϕ for the left (sub)figure in Figure 4.10, and also for each of the three situations where exactly one driver changes their path. (Note that equation (4.5) has pretty dense notation; this example is meant to encourage you to work through that notation, even if it takes some effort.)
2. Let's go back to the shown in Figure 4.10, but now we will use the (simpler) cost function given by the caption in that figure. Show that, if $c(3) > 2c(2)$, then the figure on the right is a Nash equilibrium.

B: Warmup exercises. For you to present in class. Due by end of class Wed., 3 Apr.

Exercise 4.17