Math 5370 **GAME THEORY** Spring 2019

Dr. Duval **Homework**

Wednesday, March 13

Follow the separate general guidelines for Parts A,B,C. Be sure to include and label *all four* standard parts (a), (b), (c), (d) of Part A in what you hand in.

General-sum games: Examples

Section 4.1

A: Reading questions. Due by 2pm, Sun., 24 Mar.

1. Consider a generalized version of Prisoner's Dilemma, where the cost of different situations is given by:
   (a) if one prisoner confesses but the other prisoner does not, then the prisoner who confesses gets x years ($x = 10$ in Example 4.1.1), and the prisoner who does not still goes free;
   (b) if both prisoners confess, then both prisoners get y years ($y = 8$ in Example 4.1.1);
   (c) if neither prisoner confesses, then both players get z years ($z = 1$ in Example 4.1.1).

   What conditions do we need on x, y, z for the story to make sense? Do these conditions guarantee the interesting phenomenon that confessing is a dominant strategy for each individual prisoner, yet both prisoners do better if neither player confesses than if both players confess? If not, what additional conditions do we need to assume to guarantee this phenomenon?
2. Explain why H is the unique safety strategy in Stag Hunt.
3. Verify the details of the mixed Nash equilibrium in Stag Hunt.
4. Verify that Driver and Parking Inspector has no Nash equilibrium in which either player uses a pure strategy, as the textbook claims.

B: Warmup exercises. For you to present in class. Due by the end of class Mon., 25 Mar.

Exercise 4.1

Nash equilibria
Section 4.2

A: Reading questions. Due by 2pm, Tue., 26 Mar.

1. How are Nash equilibria in general-sum games different than Nash equilibria in zero-sum games? Illustrate the differences with the examples in Section 4.1.
2. Verify that if $s < \ell < 2s$ in Cheetahs and Antelopes, then there are two pure Nash equilibria.
3. In the game of Chicken, the textbook assigns a variable value, $-M$, to the outcome that neither player swerves. This lets the authors discuss the changing payoffs and outcomes as M (the penalty for crashing) increases. Let's do something similar, but setting the value of "winning" the game (driving on, while the other player swerves) to be variable, say $1 + d$ (so in the textbook, $d = 1$, making $1 + d = 2$). Describe what happens to the game as d decreases from 1 to 0. Interpret this in terms of how outcomes will change as attitudes about "winning" this game change. What happens when $d < 0$?

B: Warmup exercises. For you to present in class. Due by end of class Wed., 27 Mar.

Change Cheetahs and Antelopes so that the cheetah I is dominant and cheetah II is submissive, so that if they both chase (and capture) the same antelope, cheetah I gets 2/3 of the shared antelope, and cheetah II gets 1/3 of the shared antelope. Find all Nash equilibria. For each Nash equilibrium, determine whether or not it is stable. (Your answer will depend on the relative values of s and ℓ.)