Math 5370 **GAME THEORY** Spring 2019
Dr. Duval **Homework**
Wednesday, April 3–preliminary version

Follow the separate general guidelines for Parts A,B,C. Be sure to include and label *all four* standard parts (a), (b), (c), (d) of Part A in what you hand in.

Transferable utility games; and the core

Sections 12.1, 12.2

A: Reading questions. Due by 3pm, Mon., 8 Apr.

1. Describe in your own words what is a transferable utility multiplayer cooperative game. Why do we consider cooperative games only if they are multiplayer? In other words, why not consider cooperative games that are only two players? What advantages do you see in restricting attention to transferable utility games?
2. Explain the motivation behind each of the two conditions on the characteristic function v of a cooperative game. Why is this function enough to characterize a cooperative game? (In other words, two different games will be considered the same if they have the same characteristic function.)
3. Explain in a little more detail each of the two conditions on allocation vector $\psi = \psi(v)$ to be in the core.
4. In Example 12.2.1 (Miners and Gold) explain why, if the total number of miners is even, then the vector $\psi = (1/2, \ldots, 1/2)$ is in the core.

B: Warmup exercises. For you to present in class. Due by the end of class Mon., 8 Apr.

Find the core of this generalization of the Glove Market: Now there are two people each with one left glove, and three other people each with one right glove.

Shapley's axioms

subsection 12.3.1

A: Reading questions. Due by 2pm, Tue., 9 Apr.

1. Although the textbook says the first three of Shapley's axioms are self-explanatory, explain them (a little bit each) anyway. (I think axiom (3), Efficiency, requires at least a little more thought than "self-explanatory" would suggest.)
2. Near the end of Example 12.3.1 (The S-veto game), the text claims, "Similarly, we can derive that $\psi_i(cw_S) = c\psi_i(w_S)$ for any $c \in [0, \infty)$. Note that to derive this, we did not use the additivity axiom." Verify this claim (without using additivity, which won't be so helpful if c isn't an integer).
3. Give some more detail explaining the equation in "Glove Market, again" on p. 197

$$v(S) = w_{12}(S) + w_{13}(S) - w_{123}(S).$$

If your explanation relies on the previous displayed equation in the book (for $u \vee w$), then also give some more explanation for that equation.

4. Fill in the missing details of the calculations in Example 12.3.3 (Four Stockholders). (To solve for v, it may help to read the proof of Lemma 12.3.2.)

B: Warmup exercises. For you to present in class. Due by end of class Wed., 10 Apr.

Exercise 12.a