Math 5370 | GAME THEORY | Spring 2019
Dr. Duval | **Main Exercises 3**

due 2pm, Thursday, February 21

1. In this problem you will analyze a 2×2 zero-sum game in almost complete generality. Assume the associated matrix is

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}.$$

To simplify the situation a little, we will assume that a, b, c, and d are all distinct, i.e., no two of these four quantities equal each other. By symmetry, we may as well assume that $a > b$. (Explain why we can make that assumption.)

Now consider two cases:

(i) $b < d$

(ii) $b > d$

In each case, give the value of the game.

2. Find the value of the following zero-sum game:

$$\begin{pmatrix} 1 & 9 \\ 2 & 7 \\ 3 & 6 \\ 6 & 4 \end{pmatrix}$$

3. (a) Let J denote the matrix with m rows and n columns, whose every entry is 1. Prove that if $\mathbf{x} \in \Delta_m$ and $\mathbf{y} \in \Delta_n$, then $\mathbf{x}^T J \mathbf{y} = 1$.

(b) Assume zero-sum games G and H each have m strategies for player I (who chooses the rows) and n strategies for player II (who choose the columns). For any strategy pair (i, j) (in other words, player I chooses strategy i and player II chooses strategy j), denote the payoff (for player I) in game G by $g_{i,j}$ and denote the payoff (for player I) in game H by $h_{i,j}$. Let b be a real number, and assume that $h_{i,j} = g_{i,j} - b$ for all i, j.

Use the result in part (a) to prove that $V(H) = V(G) - b$, where $V(G)$ and $V(H)$ denote the values of games G and H, respectively.