Math 5370 | **ALGEBRAIC STRUCTURES** | Spring 2018

Dr. Duval

Homework 7

due Thursday, March 22

Define

$$D = \left\{ \begin{pmatrix} a & -b \\ b & a \end{pmatrix} : a \in \mathbf{Z},\ b \in \mathbf{Z} \right\} \subseteq M_2(\mathbf{Z}),$$

a subset of $M_2(\mathbf{Z})$ (the set of 2×2 integer matrices).

1. Prove that D is a **subring** of $M_2(\mathbf{Z})$, with the usual matrix addition and multiplication.

2. Prove that D is an integral domain.

3. Prove that the field of quotients of D is isomorphic to

$$F = \left\{ \begin{pmatrix} p & -q \\ q & p \end{pmatrix} : p \in \mathbf{Q},\ q \in \mathbf{Q} \right\}.$$

Hints: A good first step will be to find formulas for the sum of two arbitrary matrices in D and for the product of two arbitrary matrices in D. For problem **3.**, show that every equivalence class $[M, N]$ for $M, N \in D$ contains a representative of the form (P, I) for some $P \in D$; in other words, that every pair (M, N) is equivalent to a pair of the form (P, I).