Math 5370 | **ALGEBRAIC STRUCTURES** | Spring 2018

Dr. Duval | **Homework 6**

due Thursday, March 1

1. Let $G = D_4$ be the group of symmetries of a square, described in Section 4.1, Example 12. (It is described there both geometrically, and in terms of permutations. You may use whichever description you prefer.)

(a) Prove that $H = \{e, \alpha^2\}$ is a normal subgroup of G.

(b) Find the group multiplication table of the quotient group G/H.

(c) We know that G/H has $8/2 = 4$ elements. From Section 4.4, Example 6, we know that there are only two different groups of order 4 (up to isomorphism): the cyclic group of order 4; and the **Klein four group**. Which of these two groups is G/H isomorphic to?

2. On Homework 4, Problem 1, you showed that the map $\phi\colon \mathbf{Z}_{14} \to \mathbf{Z}_7$ defined by

$$\phi([x]_{14}) = [3x]_7$$

is a an epimorphism (surjective homomorphism) from the additive group $G = \mathbf{Z}_{14}$ to the additive group $H = \mathbf{Z}_7$. You also found the kernel K of that map. Show directly (without relying on Theorem 4.27 or 4.28) that G/K is isomorphic to H, thus illustrating Theorem 4.28.