Math 5370 | **ALGEBRAIC STRUCTURES** | Spring 2018
Dr. Duval | **Homework 5**
due Thursday, February 22

1. Recall (or recreate) the multiplication table you made for the group of symmetries of a triangle. Prove that this group is isomorphic to the group of all permutations on the set $\{1, 2, 3\}$.

2. Consider the additive group of reals $\mathbf{R}$, and its subgroup of integers $\mathbf{Z}$. Describe the coset $(23/17) + \mathbf{Z}$ and the coset $5 + \mathbf{Z}$; in each case list several members of the coset, and also describe the general member of the coset. Then describe the set of all (left) cosets of $\mathbf{Z}$ in $\mathbf{R}$.

3. Let G be a group, and let $b \in G$. Define $\phi_b \colon G \to G$ by

$$\phi_b(g) = b^{-1}gb.$$

Prove that ϕ_b is an isomorphism from G to itself.

4. Let G be a group, and let H be a subgroup of G. Prove that $c \in H$ if and only if $cH = H$ and $Hc = H$.