Math 5370 | **ALGEBRAIC STRUCTURES** | Spring 2018

Dr. Duval

Homework 4

due Thursday, February 15

1. Consider the additive groups $\mathbf{Z}_{14}$ and $\mathbf{Z}_7$, and define $\phi\colon \mathbf{Z}_{14} \to \mathbf{Z}_7$ by

$$\phi([x]_{14}) = [3x]_7.$$

Prove that ϕ is a homomorphism and find $\ker \phi$. Is ϕ an epimorphism? Is ϕ a monomorphism?

2. Recall that the magnitude $|a+bi|$ of a complex number $a+bi$ is given by $|a+bi| = \sqrt{a^2+b^2}$.

(a) Prove that $\phi(a+bi) = |a+bi|$ is a homomorphism from the multiplicative group of the complex numbers $\mathbf{C}$ to the multiplicative group of positive real numbers $\mathbf{R}^+$.

(b) Draw the location in the complex plane of all the elements of $\ker \phi$.

3. Recall that if $\phi\colon G \to H$ is a bijection from set G to set H, then there is an inverse function $\psi\colon H \to G$, meaning $\psi(\phi(g)) = g$ for all $g \in G$ and $\phi(\psi(h)) = h$ for all $h \in H$. Prove that if ϕ is an isomorphism from group G to group H, then its inverse ψ is an isomorphism from H to G.