Math 5370 | **ALGEBRAIC STRUCTURES** | Spring 2018

Dr. Duval

**Homework 3**

due Thursday, February 8

**1.** Prove that $\{[1],[3],[5],[7]\}$ is a subgroup of the multiplicative group $\mathbf{Z}_8$, but is **not** a **cyclic** subgroup.

**2.** Define a mapping $\phi\colon \mathbf{Z}_n \to \mathbf{Z}_n$ by $\phi([a]) = [n-a]$, for $0 \le a < n$. Prove that $\phi$ is an isomorphism from the additive group $\mathbf{Z}_n$ to itself.

**3.** Prove that $\{a+bi\colon 5a = 7b\}$ is a cyclic subgroup of the additive group $\mathbf{C}$.

**4.** Fix a positive integer $n$. For each integer $k$, define

$$z_k = \cos(2k\pi/n) + i\sin(2k\pi/n) \in \mathbf{C},$$

and let

$$A_n = \{z_k\colon 0 \le k < n\}.$$

Prove that $A_n$ is a subgroup of the **multiplicative** group $\mathbf{C}$, and that the mapping $\phi\colon A_n \to \mathbf{Z}_n$, given by

$$\phi(z_k) = [k]$$

is an isomorphism from the **multiplicative** group $A_n$ to the **additive** group $\mathbf{Z}_n$.