Math 5370 **ALGEBRAIC STRUCTURES** Spring 2018
Dr. Duval **Homework 13**
due Thursday, May 3

1. Write

$$6x^4 - 5x^3 - 15x^2 - 21x - 10$$

as a product of its leading coefficient and a finite number of monic irreducible polynomials over the field of rational numbers.

2. Prove that

$$3x^4 + 2x^3 - 4x^2 + 6$$

is irreducible over the field of rational numbers.

3. Prove that

$$2x^3 - 5x - 1$$

is irreducible over the field of rational numbers.

4. Let $f(x) \in \mathbf{Z}[x]$. Prove that $f(x)$ cannot have **exactly** one irrational root. (Take into account multiplicity: For instance, consider $(x+1)^2$ to have **two** roots, $r = -1$ and $r = -1$.)