Math 5370 | **ALGEBRAIC STRUCTURES** | Spring 2018

Dr. Duval | **Homework 12**

due Thursday, April 26

1. Find $s(x)$ and $t(x)$ in $\mathbf{Z}_5[x]$ such that $d(x) = f(x)s(x) + g(x)t(x)$, where

$$f(x) = 2x^3 + 3x^2 + 4x + 3, \quad g(x) = x^5 + 4x^4 + 4x^3 + 3x^2 + 4x + 2.$$

2. Write

$$4x^4 + 2x^3 + 2x + 1$$

as a product of its leading coefficient and a finite number of monic polynomials over $\mathbf{Z}_5$.

3. Let F be a field. Prove that if 1 and -1 are each roots of

$$f(x) = a_0 + a_1x + a_2x^2 + \cdots a_nx^n \in F[x],$$

then

$$\sum_{i:\, i \text{ even}} a_i = \sum_{j:\, j \text{ odd}} a_j = 0.$$

4. Let D be an integral domain. By Corollary 8.8, then $D[x]$ is an integral domain. Assume that D is an ordered integral domain, with positive elements D^+, and let $c \in D$. Prove that $D[x]$ is an ordered integral domain with each of the following sets of positive elements:

(a) $D[x]^+ = \{f(x) \in D[x] \colon f(c) \in D^+\}$;

(b) $D^{[}x]^+ = \{f(x) = a_0 + a_1x + \cdots a_nx^n \in D[x] \colon a_n \in D^+\}$.

(Recall the definition of $f(c)$ from Homework 11.) Note that in part (b), n is **not** fixed; in other words, a_n there is just the leading coefficient of the polynomial.

As in Homework 8, these are two separate but somewhat similar problems.