Math 5370 | **ALGEBRAIC STRUCTURES** | Spring 2018
Dr. Duval | **Homework 11**
due Thursday, April 19

Let R be a commutative ring with identity. Let $s \in R$ and $p(x) \in R[x]$ so

$$p(x) = \sum_{i=0}^{n} a_i x^i$$

for some integer n and some $a_i \in R$. Define

$$p(s) = \sum_{i=0}^{n} a_i s^i.$$

1. Let $s \in R$. Prove that the map $\theta \colon R[x] \to R$ given by $\theta(p(x)) = p(s)$ is a surjective ring homomorphism.

2. Let $s, c \in R$. Prove that $I = \{p(x) \in R[x] \colon p(s) = c\}$ is an ideal in $R[x]$ if and only if $c = 0$.

3. Let $S \subseteq R$. Prove that $J = \{p(x) \in R[x] \colon p(s) = 0 \text{ for all } s \in S\}$ is an ideal in $R[x]$.

4. Find a counterexample to the following claim: If $S \subseteq R$, then $K = \{p(x) \in R[x] \colon p(s) = 0 \text{ for some } s \in S\}$ is an ideal in $R[x]$.