Math 5370 **ALGEBRAIC STRUCTURES** Spring 2018
Dr. Duval **Homework 10**
due Thursday, April 12

1. Let $\theta\colon R \to S$ be a surjective ring homomorphism. Prove that if $\operatorname{char} S > 0$, then $\operatorname{char} R \leq \operatorname{char} S$.

2. Recall from Homework 9 Problem 1 that if $k \in \mathbf{Z}$ then

$$I = \{a + b\sqrt{2}\colon a, b \in k\mathbf{Z}\}$$

is an ideal in

$$R = \{a + b\sqrt{2}\colon a, b \in \mathbf{Z}\}.$$

Let $k = 2$.

(a) List the distinct cosets of R/I. [Here, "distinct" means list each coset exactly once, even though the same coset may have different names.]

(b) Write down the addition and multiplication tables of R/I.

(c) Is R/I a field? Justify your answer.

3. Repeat Problem **2.** above, but with $k = 3$ instead of $k = 2$.