Math 5370 **ALGEBRAIC STRUCTURES** Fall 2016
Dr. Duval **Homework**
Thursday, November 17

Written homework. Due in writing, at the beginning of class,

Thursday, December 1. 6.26, 6.29, 6.37, 6.43.

Warmup exercises. To present in class.

Tuesday, November 29. 6.42.

Thursday, December 1. 7.2, 7.3, 7.4(ii, iv, vi), 7.12.

Reading assignment. These reading questions cover Section 7.1 on Quotient Rings. If all goes well, we will spend most of our time on Thursday discussing the warmup problems for that day. (So try to have all the reading questions ready by Tuesday.)

1. Illustrate Proposition 7.4 in the case where $R = \mathbb{R}[x]$, and $I = (x^2 + 1)$.
2. In Lemma 7.6, what does "well-defined" mean?
3. Theorem 7.10 is an important theorem, though there is some notation to wade through. To help you start to penetrate what the theorem is saying, let's identify some of the objects mentioned there in the case when $R = \mathbb{Z}$, $A = \mathbb{Z}_6$, and ϕ is the usual map $\phi\colon \mathbb{Z} \to \mathbb{Z}_6$. What is $\ker \phi$? What is $\operatorname{im} \phi$? Show $\ker \phi$ is an ideal of $\mathbb{Z}$. Show $\operatorname{im} \phi$ is a subring of $\mathbb{Z}_6$. Can you make sense of the statement

$$R/\ker \phi \cong \operatorname{im} \phi$$

in this case?