Math 4370 **COMBINATORICS** Spring 2012
Dr. Duval **Homework**
Thursday, April 19

Follow the separate general guidelines for Parts A,B,C. Be sure to include and label *all four* standard parts (a), (b), (c), (d) of Part A in what you hand in.

Graph Isomorphisms, part 1

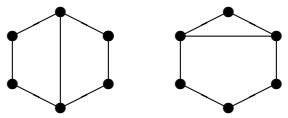
Section 5.2, up through and including Example 5.6

A: Reading questions. Due by 3pm, Mon., 23 Apr.

1. Explain carefully the claim in the caption of Figure 5.7, that "the two graphs are identical as labeled graphs, even if they look different." In what way do the graphs look different (OK, that's pretty easy, but answer it anyway). Then (a little harder), why are they identical as labeled graphs? The definition asks you to check something for all pairs of vertices; explicitly check this for several different pairs of vertices.
2. Why are the graphs in Figure 5.8 not identical as labeled graphs? Are they isomorphic? Why or why not?
3. Draw your own example of graphs that look different, but are actually isomorphic. Give an explicit description of the isomorphism between your two graphs.
4. Describe the two automorphisms of the graph J in Figure 5.9. Explain why each one is an automorphism. (You may need to put labels on J, just to describe the automorphisms.)
5. Illustrate the definition of $l(J)$ by listing all the labelings of the graph G in Figure 5.8 that are counted by $l(G)$.

B: Warmup exercises. For you to present in class. Due by the end of class Tue., 24 Apr.

1. Are the following two connected graphs isomorphic? Explain why or why not.



2. Find all different (non-isomorphic) unlabeled graphs with 4 vertices. (The graphs do **not** need to be connected.) [Hint: There are between 10 and 15 such graphs.]
3. Pick two of your graphs with 4 vertices and 3 edges, and find all the different (non-isomorphic) labelings of each of those graphs.

Graph Isomorphisms, part 2

Section 5.2, starting with paragraph above Theorem 5.7

A: Reading questions. Due by 3pm, Wed., 25 Apr.

1. Explain how the graph in Figure 5.8 satisfies Theorem 5.7.
2. In Example 5.8, the author claims $l(H) = 3$ and $|Aut(H)| = 2$. Explain why each of these equations is true.
3. Draw the graph H described in Example 5.9.
4. Show several (at least 3) of the automorphisms of the the graph H described in Example 5.9.

B: Warmup exercises. For you to present in class. Due by the end of class Thu., 26 Apr.

1. **5.12 Supplementary Exercise:** 12; also find $|Aut(G)|$.