Math 4370 Dr. Duval

COMBINATORICS Homework

Tuesday, April 3

Follow the separate general guidelines for Parts A,B,C. Be sure to include and label *all* four standard parts (a), (b), (c), (d) of Part A in what you hand in.

Products of ordinary generating functions, part 1

Subsection 3.3.1 (up through and including Example 3.15)

A: Reading questions. Due by 3pm, Wed., 4 Apr.

- 1. When (i.e., for what kind of counting problem) do we multiply generating functions? Describe how each of the first three examples (Examples 3.12, 3.13, 3.15) in this section fit this pattern (i.e., why we multiply generating functions in each example). Note that you might want to not answer this question first, but at least keep it in mind as you read through the section, as this is the central idea of the section.
- 2. In the solution to Example 3.12, why is $B(x) = \frac{1}{1-3x}$ and $D(x) = \frac{x}{(1-x)^2}$.
- 3. In the solution to Example 3.13, how does the author get from the last expression on the bottom of p. 144 (formula for A(x) with five terms) to the first expression at the top of p. 145 (five summations)?
- 4. In the solution to Example 3.15, why does it suffice to find $[x^{n-5}]\frac{1}{(1-x^2)(1-x)^2}$?

B: Warmup exercises. For you to present in class. Due by the end of class Thu., 5 Apr.

1. Define a sequence a_0, a_1, a_2, \ldots by $a_0 = 1$ and $a_n = 1 + \sum_{i=1}^{n-1} (n-i)a_i$ for $n \ge 1$. Find the generating function for this sequence.