Math 4370 | **COMBINATORICS** | Spring 2012

Dr. Duval

Homework

Thursday, March 22

Follow the separate general guidelines for Parts A,B,C. Be sure to include and label *all four* standard parts (a), (b), (c), (d) of Part A in what you hand in.

Power series

Section 3.1

A: Reading questions. Due by 3pm, Wed., 28 Mar.

1. How is a power series the same as a polynomial, and how is it different?
2. Explain how Definition 3.1 matches our usual definition of $\binom{a}{k}$ when a is an integer.
3. Show how Theorem 3.2 is a generalization of the Binomial Theorem (Theorem 1.24). In other words, show how Theorem 3.2 becomes Theorem 1.24 in the special case where a is a positive integer.
4. How is a formal power series the same as a power series, and how is it different?
5. To help make sense of the displayed equations near the top of p. 128 for $(A+B)(x)$ and especially $(A \cdot B)(x)$, let's try it with finite polynomials, instead of (infinite) power series. Consider the special case where $a_n = 0$ for $n \geq 5$ and $b_n = 0$ for $n \geq 5$. Write out what $A(x)$ and $B(x)$ are in this case (in terms of a_i's and b_i's). Use rules of polynomials to verify the formulas for $(A+B)(x)$ and $(A \cdot B)(x)$.
6. Which parts of this section would you have found (or did find) helpful in reading subsection 3.2.1? Would you rather that we'd covered this section before starting subsection 3.2.1?

B: Warmup exercises. For you to present in class. Due by the end of class Thu., 29 Mar.

1. Use finite polynomials to justify the equations for the derivative and integral of a formal power series.
2. Check the details of the two different proofs of equation (3.4). [Note: This equation was in the publicity poster for the course.]

Ordinary generating functions, part 2

Subsection 3.2.1 (Example 3.6 through Example 3.7)

A: Reading questions. Due by 3pm, Mon., 2 Apr.

1. In Example 3.6, the author states that finding the coefficient of x^n in $F(x)$ is equivalent to finding the coefficient of x^{n-1} in $\frac{1}{1-3x}$. Explain why this is so.
2. Justify both of the equalities in the displayed equation in the solution to Example 3.6.
3. Explain in your own words how, on the bottom of p. 132, we get from

$$\sum_{n\geq 1} a_n x^n = 3\sum_{n\geq 1} a_{n-1}x^n - \sum_{n\geq 1} x^n$$

to

$$A(x) - 5 = 3xA(x) - \frac{x}{1-x}.$$

4. Verify directly that the solution in equation (3.7) satisfies the red hat recurrence relation at the beginning of the section.
5. The summations in the displayed equations near the top of p. 136 have summation indices "$n \geq 2$". Why? How does this affect the derivation of the first relation for $B(x)$ (the equation that starts with $B(x) - 15x - 5$)?

B: Warmup exercises. For you to present in class. Due by the end of class Tue., 3 Apr.

1. Use generating functions to solve the recurrence relation $a_n = 5a_{n-1} + 2$, with initial condition $a_0 = 2$.
2. Use generating functions to solve the recurrence relation $b_n = 5b_{n-1} - 6b_{n-2}$ with initial conditions $b_0 = 0$, $b_1 = 1$.
3. Use generating functions to find a closed formula for the Fibonacci numbers, $0, 1, 1, 2, 3, 5, 8, \ldots$ ($f_0 = 0$, $f_1 = 1$, $f_n = f_{n-1} + f_{n-2}$).