Math 4370 **COMBINATORICS** Spring 2012
Dr. Duval **Homework**
Thursday, February 9

Follow the separate general guidelines for Parts A,B,C. Be sure to include and label *all four* standard parts (a), (b), (c), (d) of Part A in what you hand in.

Recurrence relations for Stirling numbers of the second kind
Subsection 2.2.2 [up through and including the first paragraph of p. 68]

A: Reading questions. Due by 3pm, Wed., 15 Feb.

1. Demonstrate the bijection in the proof of Theorem 2.10 when $n = 4$ and $k = 2$. Note that $S(4, 2) = 7$, so each of the sets in your bijection should have 7 elements.
2. Use Theorem 2.10 to generate the next $(n = 6)$ row of Figure 2.2.
3. Repeat Examples 2.12 and 2.13 with $n = 5$ and $k = 2$.
4. Illustrate the proof of Lemma 2.14 with $n = 5$ and $k = 2$. (Show how each statement in the proof works for these values of n and k. This resembles Example 2.15.)
5. What are the similarities and differences between the recurrence for Stirling numbers of the second kind in Theorem 2.10 and the recurrence for binomial coefficients in Theorem 1.36?

B: Warmup exercises. For you to present in class. Due by end of class Thu., 16 Feb.

1. **2.10 Supplementary Exercise:** 13

Set partitions, continued

Subsections 2.2.2 [starting from second paragraph of p. 68] and 2.2.3

A: Reading questions. Due by 3pm, Mon., 20 Feb.

1. Use Theorem 2.16 (and Figure 2.2) to show how to compute $S(5,3) = 25$.
2. Now, illustrate the proof of Theorem 2.16 on $n = 4$ and $k = 3$. In other words, the proof works by showing two different ways to enumerate partitions of $[n+1]$ into k blocks. The first way is simply the definition of Stirling numbers of the second kind, $S(n+1,k)$, which is $S(5,3)$ in this case, enumerating partitions of [5] into 3 blocks. The second way counts all these partitions piece by piece (for different values of i). Find the partitions corresponding to each value of i, and show why they are counted by $\binom{n}{i}S(n-i,k-1)$.
3. At the top of p. 69, the author invites you to explain why the proof of Theorem 2.16 works even in the special cases when $n - i = 0$ or $k - 1 = 0$. Accept this invitation, and explain this phenomenon.
4. List the partitions that show $B(4) = 15$.
5. Use Theorem 2.18 (and Figure 2.3) to show how to compute $B(5) = 52$.

B: Warmup exercises. For you to present in class. Due by the end of class Tue., 21 Feb.

1. **2.10 Supplementary Exercise:** 10 (first solve this without the restriction on Adam and Brenda).