Math 4370 — **COMBINATORICS** — Spring 2012

Dr. Duval — **Homework**

Thursday, February 2

Follow the separate general guidelines for Parts A,B,C. Be sure to include and label *all four* standard parts (a), (b), (c), (d) of Part A in what you hand in.

## Compositions

Subsection 2.1.2

**A: Reading questions.** Due by 3pm, Wed., 8 Feb.

1. What is the difference between compositions and weak compositions?
2. Demonstrate the bijection in the proof of Corollary 2.5 when n=6 and k=3. In other words, demonstrate the bijection from $W$, weak compositions (of what number into how many parts?), to $C$, compositions of 6 into 3 parts.
3. In the proof of Corollary 2.5, the author defines a function $f$ from $W$, weak compositions, to $C$, compositions, and alleges that $f$ is a bijection (the same one you demonstrated in the previous reading question). The author also writes, "The reader is asked to verify that $f$ is bijection." Accept this invitation, and show that $f$ is in fact a bijection. (Remember the 3-step process for showing a function is a bijection in Section 1.4).

**B: Warmup exercises.** For you to present in class. Due by end of class Thu., 9 Feb.

1. How many different ways can you distribute 7 identical soccer balls to 3 (different) children, if each child must receive at least one soccer ball?
2. How many different ways can you distribute 12 identical soccer balls to 5 (different) children, if each child must receive at least one soccer ball? What if no child may receive eight? What if one particular child, your cousin, must receive at least two?
3. How many different ways can you distribute 18 identical soccer balls to 5 (different) children, if each must receive an even number of soccer balls? (Solve this both in the case where children may receive no soccer balls, and the case where each child must receive at least one soccer ball.)

**Stirling numbers of the second kind**
Subsection 2.2.1

**A: Reading questions.** Due by 3pm, Mon., 13 Feb.

1. In the example that leads off this section (occupying most of p. 63), are the rooms allowed to be empty, or not?
2. In the same example as the previous question, the author suggests there are two reasonable interpretations of the initial question, and states that the answers to these two interpretations differs by a factor of $3! = 6$. Show this by finding one solution to the problem when the rooms are the same, and the $3! = 6$ corresponding solutions to the problem when the rooms are different.
3. Example 2.8 describes how to count the partitions of [5] into three blocks. List all of these partitions, and show how each one matches the description given in Example 2.8.
4. Near the end of this subsection, on p. 65, the author writes, "The reader is asked to verify that for all positive integers $n$, we have $S(n, 1) = S(n, n) = 1$." Accept this invitation, and prove that $S(n, 1) = 1$ and $S(n, n) = 1$.

**B: Warmup exercises.** For you to present in class. Due by the end of class Tue., 14 Feb.

1. List all the ways of distributing 4 different objects into 2 identical boxes.
2. List all the ways of distributing 5 different objects into 2 identical boxes.
3. List all the ways of distributing 5 different objects into 4 identical boxes.
4. List as many ways as you can of distributing 6 different objects into 4 identical boxes; we'll try to come up with the complete list in class.
5. Make a table with as many entries of $S(n, k)$ as you can, using just the examples, reading questions, and warmup exercises in this subsection.