Math 4370 | **COMBINATORICS** | Spring 2012

Dr. Duval | **Homework**

Thursday, January 26

Follow the separate general guidelines for Parts A,B,C. Be sure to include and label *all four* standard parts (a), (b), (c), (d) of Part A in what you hand in.

Pigeonhole principle

Section 1.5

A: Reading questions. Due by 3pm, Wed., 1 Feb.

1. The author rephrases Theorem 1.43 using boxes and balls, in the paragraph on p. 35 starting, "A classic way of thinking about the Pigeonhole Principle…". Carefully match up the details of this paragraph to Theorem 1.43, so a classmate would understand and believe that the theorem and the "classic way" are the same thing.
2. In the middle of p. 35, the author suggests that putting pigeons into holes is inhumane. In light of this, can you think of a better name for the idea of this section than "pigeonhole principle"? (Don't spend too much time on this question.)
3. Rework Example 1.44, except use "Texas" for "New York City". Make your statement as strong as possible. (You may need to look up some population data; use the 2010 census.)
4. Near the bottom of p. 36, the author states, "A frequently applied special case of the Pigeonhole Principle is when $r = 1$." Restate Theorem 1.43 in this special case.

B: Warmup exercises. For you to present in class. Due by end of class Thu., 2 Feb.

1. **1.10 Supplementary Exercise:** 38
2. **1.10 Supplementary Exercise:** 39
3. In Example 1.46(a), can $2n+1$ be replaced by $2n$? Why or why not? In Example 1.46(b), can n be replaced by $n+1$? Why or why not?

Weak compositions
Subsection 2.1.1

A: Reading questions. Due by 3pm, Mon., 6 Feb.

1. In the example that leads off this section (about the movie-watching family answering the phone and eating ice cream), at one point (second paragraph of p. 59) the author says "the *order* in which the four people pick up the phone does not matter". Later (near the top of p. 60), the author says, "Note that the *order of the summands* matters". Explain how this apparent discrepancy (it seems that order both matters and doesn't matter) is possible.
2. Just after Definition 2.1, the author says "there is a natural bijection from the set of weak compositions of $[n]$ into k parts, and n-element multisets over a k-element set." Demonstrate this bijection on the weak composition of 17 into 5 parts, $(3, 0, 5, 2, 7)$.
3. Demonstrate the bijection at the heart of the proof of Theorem 2.2 on the same weak composition in the previous question, $(3, 0, 5, 2, 7)$.

B: Warmup exercises. For you to present in class. Due by the end of class Tue., 7 Feb.

1. How many different ways can you distribute 7 identical soccer balls to 3 (different) children? (There is no restriction on the number of soccer balls each child may receive. Some children may receive no soccer balls. One child could get them all.)
2. I want to make fruit baskets containing exactly 9 fruits, and I have a supply (essentially unlimited) of bananas and oranges. How many different fruit baskets can I make, if I only care about the *number* of pieces of each different fruit? Now answer this question if I have a supply (again, unlimited) of apples, bananas, and oranges.
3. A department of 13 workers needs to form four committees, and no person may serve on more than one committee. The Walrus committee and the Xylophone committee each need 3 members, the Yak committee needs only 2 members, and the Zither committee needs 5 members. In how many ways can these committees be formed? What if there are, instead, 15 members in the department?