Math 4326/5322 **LINEAR ALGEBRA** Spring 2019
Dr. Duval **Homework**
Monday, April 29

Follow the separate general guidelines for Parts A,B,C. Be sure to include and label *all four* standard parts (a), (b), (c), (d) of Part A in what you hand in.

Positive Operators and Isometries (Part I): Positive Operators

Section 7.C, pp. 225–227

A: Reading questions. Due by 2pm, Sun., 5 May

1. Verify Example 7.32(a).
2. The hardest part of the proof of result 7.35 (Characterization of positive operators) is (b) $\Rightarrow$ (c), but it's not that hard. Where in that part of the proof do we use the hypothesis that T is self-adjoint, and where do we use the hypothesis that all of the eigenvalues of T are nonnegative?
3. Find three different examples of positive operators in $\mathcal{L}(\mathbf{R}^2)$. (Hint: Can you use any part of result 7.35?)
4. In the side note at the beginning of the proof of result 7.36 (Each positive operator has only one positive square root), the textbook claims that the identity operator on V has infinitely many square roots if $\dim V > 1$. Verify this for the case $V = \mathbf{R}^2$ by finding an infinite family of square roots of I. (Hint: Find solutions to the equation $R^2 = I$ by assigning variables to all the entries of the matrix corresponding to R, and then find solutions to the resulting system of equations.) You do **not** have to find **all** the solutions.

B: Warmup exercises. For you to present in class. Due by the end of class Mon., 6 May

Exercises 7.C: 5

Positive Operators and Isometries (Part II): Isometries

Section 7.C, pp. 228–231

A: Reading questions. Due by 2pm, Tue., 7 May

1. The short paragraph between the definition of isometry (7.37) and Example 7.38 claims that "we will soon see that if $\mathbf{F} = \mathbf{C}$, the the next example includes all isometries." When do we see this?
2. The paragraph above result 7.42 (Characterization of isometries) claims that an operator is an isometry "if and only the list of columns of its matrix with respect to ... some basis is orthonormal". Verify this in the case of the orthonormal basis of $\mathbf{R}^2$, $((3/5, 4/5), (-4/5, 3/5))$. In other words, show that if we make a matrix M whose columns are the two vectors of that basis, then the operator S corresponding to M an isometry. (It is probably easier to do this working with M instead of S, i.e., using the matrix directly.)

3. Continuing the previous question: Verify that S from that question satisfies parts (b) and (h) of result 7.42. (Once again, this may be easier working with M directly.)
4. The paragraph above result 7.43 (Description of isometries when $\mathbf{F} = \mathbf{C}$) says that we need to know that isometries are normal in order to prove result 7.43. Where in that proof do we actually use that S is normal?
5. What will you do with all the time you have, now that there are no more reading questions to answer?

B: Warmup exercises. For you to present in class. Due by end of class Wed., 8 May

Exercises 7.C: 10