Math 4326/5322 | **LINEAR ALGEBRA** | Spring 2019

Dr. Duval

Homework

Monday, April 8

Follow the separate general guidelines for Parts A,B,C. Be sure to include and label *all four* standard parts (a), (b), (c), (d) of Part A in what you hand in.

Orthonormal Bases (Part I): Gram-Schmidt

Section 6.B, pp. 180–185

A: Reading questions. Due by 2pm, Sun., 14 Apr.

1. Verify the lists in Example 6.24 are indeed orthonormal, as claimed in the text.
2. Demonstrate result 6.30 (Writing a vector...) with $V = \mathbf{F}^3$, orthonormal basis (e_1, e_2, e_3) given by the list in Example 6.24(c), and $v = (9, 1, 5)$.
3. Try to read the proof of the Gram-Schmidt Procedure (result 6.31) without worrying too much about the precise algebraic details of the equation defining e_j or the calculation in the middle of p. 183. The second sentence of the statement of the result says, "For $j = 2, \ldots, m$, define e_j inductively...". What, in your own words, does that mean in this case?
4. Near the bottom of p. 182, the text asks, "does $\mathcal{P}_m(\mathbf{F})$, with inner product [given by 6.4(c)] have an orthonormal basis?" Answer this question, and explain your answer. [Note: you do **not** have to produce such a basis, just decide whether or not it exists.]

B: Warmup exercises. For you to present in class. Due by the end of class Mon., 15 Apr.

Exercises 6.B: 5.

Orthonormal Bases (Part II): Schur's and Riesz's Theorems
Section 6.B, pp. 168-174

A: Reading questions. Due by 2pm, Tue., 16 Apr.

1. The proof of result 6.37 (Upper-triangular...) relies on the proof of the Gram-Schmidt Procedure (result 6.31), in particular claiming that applying the Gram-Schmidt Procedure to $v_1, \ldots, v_n$ produces an orthonormal basis $e_1, \ldots, e_n$ such that
$$\operatorname{span}(e_1, \ldots, e_j) = \operatorname{span}(v_1, \ldots, v_j)$$
for **each** j. Verify that the orthonormal basis e_1, e_2, e_3 in Example 6.33 satisfies this condition for each of $j = 1, 2, 3$.
2. The proof of the Riesz Representation Theorem (result 6.42) is not too hard, but to warm up to the idea, prove the converse (which is easier to prove): If V is an inner-product vector space, then the function ϕ_u **defined** by
$$\phi_u(v) = \langle v, u \rangle$$
is a linear functional on V.
3. In Example 6.44, we replace $\cos(\pi t)$ by $u(t)$ in the integral. Why is this worth doing? In other words, in what way is the expression with $u(t)$ better than the expression with $\cos(\pi t)$?
4. Verify the comment in the final paragraph of the section on p. 189 (about equation 6.43 in the proof of the Riesz Representation Theorem) for Example 6.40, first with the standard basis, and then with the orthonormal basis in Example 6.24(c).

B: Warmup exercises. For you to present in class. Due by end of class Wed., 17 Apr.

Prove that $\phi(p) = p(3)$ is a linear functional on $\mathcal{P}(\mathbf{R})$.